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IOANNIS CAMILLI GLORIOSI EXERCITATIONVM MATHEMATICARVM

Decas prima.

IN QVA

Continentur varia & theoremata & problemata, tum ei ad ſoluendum propoſita, tum ab eo inter legendum animaduerſa.

BIBLIOTECA NAZIONALE VITTORIO EMANUELE III NAPOLI

NEAPOLI,

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AD MATHEMATICARVM STVDIOSOS.

CVM per plures annos Patauij publice Mathematicas diſciplinas exercuerim,cumq; inſigniores Italiæ Ciuitates peragrauerim, à nobilibus viris varia mihi fuerunt propoſita & theoremata & problemata ad reſoluendum ; Accidit quoq; quod in Mathematicorum lucubrationibus perluſtrandis quamplúrima animaduerti, quæ vel ſupplemento vel correctione indigere videbantur,ideoque quædam ſuppleui & quædam correxi; nunc in patriam reuerſus ocio fruens liberali, meaque ſchediaſmata reuoluens præfatas Mathematicas ſpeculationes in ordinem redigere ſtatui, atque accuratius collimare; At quia contemplationes hæ diuerſorum ſunt generum,variaſque con-

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tinent materias, ſub vno & continuato the
mate vel titulo ordinari ac redigi non po-
tuerunt, placuit eas quapropter ſub titulo
Mathematicarum exercitationum in publi
cum emittere; ſtatueram equidem has om
nes exercitationes ſimul vno & eodem vo-
lumine complecti atque euulgare, ſed quia
omnes adamuſsim adhuc diſpoſitæ ac di-
geſtæ non ſunt, eas ad tempus opportu-
num reſeruauimus, ex his nunc decem exer
citationes ſelegimus, quas occaſione qua-
dam nobis oblata in lucem prodire ſinui-
mus; Hanc igitur noſtram Mathematica-
rum exercitationum primam decadem
Mathematicarum ſtudioſi æqui boniqueſ
conſulite.

IOAN-

IOANNIS CAMILLI GLORIOSI EXERCITATIONVM MATHEMATICARVM

Decas prima.

EXERCITATIO PRIMA.

CVM eſſem Venetijs anno 1613. ab Auguſtino Amulio Patritio Veneto fuit mihi propoſitum quęſitum Geometricum tenoris ſequentis.

Si ſemicirculi diameter producatur, & à quouis puncto productę diametri in cauam ſemicirculi periferiam ducantur duæ rectæ lineæ, num anguli à dictis rectis lineis & producta diametro comprehenſi, arcubus ſemicirculi, quibus inſiſtunt, proportionales ſint, determinare.

De qua re publice reſpondi, ibidem impreſſo libello, placuit

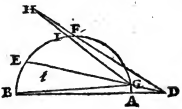
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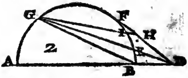
placuit nunc eandem contemplationem aggredi, & ordinatius quibuſdam additis theorematis abſoluere.

THEOREMA. I.

SI à puncto diametri extra circulum productæ in cauam vel conuexam circuli periferiam plures cadant rectæ lineæ, tamē ad eaſdem partes, æquales angulos continentes, inæquales abſcindent arcus, minorque ſemper erit ille, qui diametro proximior eſt.

Eſto circulus, cuius diameter B A, punctum aſſumptum D, ſint primo anguli æquales B D E, E D F continui, vt in duabus primis figuris, Dico arcum B E arcu E F minorem



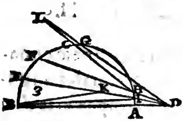


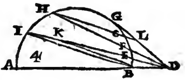
eſſe, recta E D ſecet circulum in G, cum itaque in 1. D F, minor ſit quam D B, at in 2. maior, fiat D H ipſi D B æqualis, ducanturque B G, G H, & G H ſecet circulum in I, Quare cum duo latera B D, D G æqualia ſint duobus lateribus H D, D G, anguloſque comprehendant æquales, erunt & reliqui B G D, H G D, & proinde B G E, E G I. hoc eſt arcos B E, E I inter ſe ęquales, ſed arcus E I minor eſt quàm E F, ergo arcus B E eodem E F minor erit.

Sint

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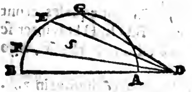
Sint ſecũdo ęquales anguli B D E, F D G nõ cõtinui, vt in 3. & 4. Dico arcum B E arcu F G minorem eſſe, rectę E D, F D ſecẽt circulum in I & H, cum itaque D G, D I in-

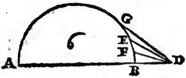




3. in mores ſint quam D B, D H, at in. 4. maiores, fiant D L, D K æquales ipſis D B, D H, ducanturque H L, B K, & H L ſecet circulum in C, Quare cum duo latera B D, D K ęqualia ſint duobus lateribus L D, D H, anguloſque comprehendant ęquales, erunt & reliqui B K D, L H D, & proinde B K E, C H F inter ſe ęquales, ſed B K E maior eſt quã B I E, ergo B I E quam C H F, hoc eſt arcus B E quam F C minor erit, ſed arcus F C minor eſt quam F G, ergo arcus B E multo magis eodem F G minor erit.

Sint tertio ęquales anguli B D E, F D G intercepti, vt in duabus vltimis figuris, Dico arcũ B E arcu F G minorẽ eſſe,





nam tunc ablato angulo communi F D E, ęquales quoque remanebunt anguli B D F, E D G, Quare ex demonſtratis minor erit arcus B F quam E G; ergo addito communi ar-

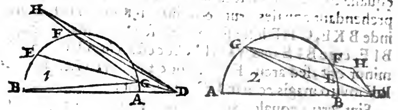
cu

cũ F E, erit arcus B E minor quam F G, quod erat demonſtrandum.

THEOREMA. II.

ET contra ſi præfatæ rectę lineæ ęquales abſcindant arcus, inæquales angulos continebunt, minorque ſemper erit ille, qui à diametro longius abeſt.

Sint primo ęquales arcus B E, E F continui, vt in duabus primis figuris, Dico angulum E D F angulo B D E minorẽ eſſe, connexis rectis lineis B G, G F, cum itaque G F minor

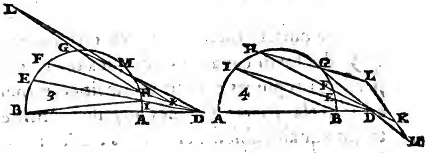


ſit quam G B, fiat G H ipſi G B ęqualis, ducaturque D H, Quare cum duo latera B G, G D ęqualia ſint duobus lateribus H G, G D, angulosque comprehendant ęquales, erunt & reliqui B D G, H D G, & proinde B D E, H D E inter ſe ęquales, ſed angulus E D F minor eſt quam H D E, ergo angulus E D F angulo B D E minor erit.

Sint ſecũdo ęquales arcus B E, F G nõ cõtinui, vt in 3. & 4. Dico angulũ F D G angulo B D E minorem eſſe, connexis rectis lineis B I, H G, cum itaque in. 3. D I minor ſit quam D H, at in. 4. maior, ſitque in vtraque G H minor quã B I, fiant H K, H L ęquales ipſis D I, B I, ducaturque K L

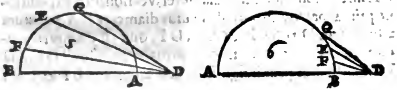
ſecans

ſecans D G in M, Quare cum duo latera D I, B I ęqualia ſint duobus lateribus K H, L H, anguloſque comprehen-



dant æquales, erunt & reliqui B D I, L K H, & proinde B D E, F K L inter ſe æquales, ſed angulus K D M hoc eſt F D G minor eſt quam F K L, ergo angulus F D G angulo B D E minor erit.

Sint tertio æquales arcus E E, F G intercepti, vt in duabus vltimis figuris, Dico angulum F D G angulo B D E minorem eſſe,

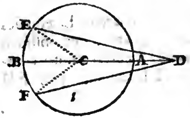


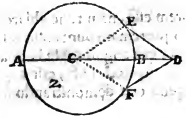
norem eſſe, nam tunc ablato communi arcu F E, æquales quoque remanebunt arcus B F, E G, Quare ex demonſtratis minor erit angulus E D G quam B D F, ergo addito communi angulo F D E, erit angulus F D G minor quam B D E, quod erat demonſtrandum.

THEOREMA. III.

SI à punĉto diametri extra circulũ productæ in cauam vel conuexam circuli periferiam plures cadant rectæ lineę, tamen ád diuerſas partes æquales angulos continẽtes, inæquales abſcindent arcus, minorque ſemper erit ille, qui diametro proximior eſt, vnico tamen excepto caſu, quando videlicet illæ rectæ lineæ vtrinque cum diametro æquales angulos fecerint, nam tunc æquales quoque abſcindent arcus.

Præſens theorema bipartitum eſt, nam vel inter has rectas lineas connumeratur diameter, vel non; Pro expeditione primæ partis, eſto circulus, cuius diameter B A, punctum aſſumptum D, rectæ lineæ D E, D F, quæ cũ diametro producta B D vtrinque contineant æquales angulos B D E, B D F, vt in duabus primis figuris, Dico arcus B E, B F æquales

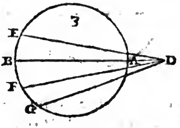


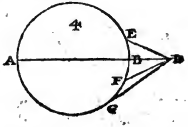


eſſe, nam tunc ductis à centro C rectis lineis C E, C F, erunt duo latera C E, C D ęqualia duobus lateribus C F, C D,

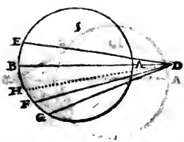
C D, ſunt & anguli E D C, F D C æqualibus lateribus C E,C F oppoſiti inter ſe æquales,nec nõ C E D,C F D vterque recto minor in 1. at in 2. vterque non minor recto, vt demonſtrabitur in ſcholio,Quare per ea,quæ notauit Clauius ad finem libri primi Element. ęquales quoque erunt rectæ lineę D E,D F, & reliqui anguli E C D,F C D,& proinde B C E,B C F, hoc eſt arcus B E,B F

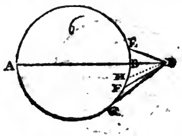
Sed ſi æquales anguli B D E,F D G non fuerint vtrinque à diametro, Dico arcum B E arcu F G minorem eſſe , nam tunc vel anguli,qui ſunt vtrinque à diametro B D E,B D F æquales fuerint,vt in 3.& 4. & tunc quoque ęquales erunt





anguli B D F,F D G, itemque arcus B E, B F, Quare ex demonſtratis minor erit arcus B F hoc eſt B E quam F G;At ſi anguli, qui ſunt vtrinque à diametro B D E, B D F ęquales

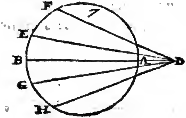


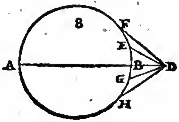


non fuerint , fueritque angulus B D E minor quam B D F , vt in 5.& 6.tunc fiat angulus B D H ęqualis angulo B D E,

& tuuc quoque ęquales erunt anguli B D H,F D G, itemq; arcus B E,B H,Quare ex demonſtratis minor erit arcus B H hoc eſt B E quam F G

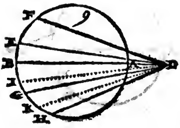
Pro expeditione ſecundę partis, quando videlicet diameter non connumeratur inter pręfatas rectas lineas, ſi ęquales anguli E D F,G D H ita conſtituti ſint, quod anguli B D E,B D G, qui ſunt vtrinque à diametro, inter ſe quoque ęquales fuerint, vt in 7. & 8. Dico arcus E F, G H

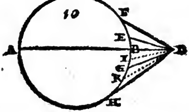




ęquales eſſe, nam tunc ex demonſtratis ęquales erunt arcus B E,B G,ergo ſi ęqualibus angulis B D E, B D G ęquales addãtur anguli E D F,G D H, ęquales quoque fient anguli B D F,B D H,& arcus B F,B H ;Quare ſi ab ęqualibus arcubus B F,B H ęquales auferantur arcus B E,B G,ęquales remanebunt arcus E F,G H

Sed ſi ęquales anguli E D F , G D H ita conſtituti ſint ,





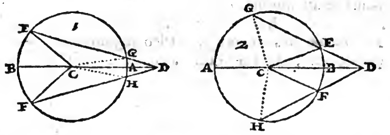
quod anguli B D E,B D G, qui ſunt vtrinque à diametro,

ęqua-

ęquales non fuerint, fueritque angulus B D E minor quam B D G, vt in vltimis duabus figuris, Dico arcum E F arcu GH minorem eſſe, nam tunc ſi fiant anguli B D I, I D K ęquales angulis B D E, E D F, ęquales quoque erunt anguli I D K, G D H, itemque arcus E F, I K, Quare ex demonſtratis arcus I K hoc eſt E F minor erit quam G H, quod erat demonſtrandum.

SCHOLIVM.

Aſſumpſimus in 1. figura angulos C E D, C F D recto minores at in 2. non recto minores, quod quidem ſatis manifeſtum eſt, nam in 1. ductis ex centro C rectis lineis C G, C H, conſtituentur duo iſoſcelia triangula C E G, C F H, ſed anguli ſupra baſim iſoſcelium triangulorum acuti ſunt, quare anguli C E G, C F H, ſiue C E D, C F D recto minores erunt.



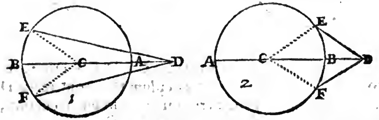
Manifeſtum quoque eſt angulos C E D, C F D in 2. recto non eſſe minores; nam ſi circulum tetigerint rectæ D E, D F, erunt præfati anguli recti, ſi vero circulũ ſecuerint, vt in noſtro exemplo, producantur ad G & H, quare ductis ex centro C rectis lineis C G, C H iſoſcelia erunt triangula C G E, C H F, angulique C E G, C F H acuti, & proinde reliqui deinceps obtuſi C E D, C F D

THEO-

THEOREMA. IV.

ET contra ſi præfatæ rectæ lineæ æquales abſcindant arcus, inæquales angulos continebunt, minorque ſemper erit ille, qui à diametro longius abeſt, vnico tamen excepto caſu, quando videlicet illæ rectæ lineæ vtrinque cum diametro æquales arcus abſcinderint, nam tunc æquales quoque continebunt angulos.

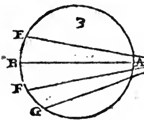
Præſens theorema bipartitum eſt, nam vel inter has rectas lineas connumeratur diameter, vel non; Pro expeditione primæ partis eſto circulus, cuius diameter B A, punctum aſſumptum D, rectę lineę D E, D F, quæ cum diametro producta B D vtrinque abſcindant ęquales arcus B E, B F, vt in duabus primis figuris, Dico angulos B D E, B D F ęquales eſſe; nam tunc ductis à centro C rectis lineis C E,

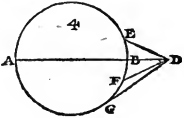


C F, erunt duo latera C E, C D æqualia duobus lateribus C F, C D, ſunt & anguli comprehenſi inter ſe æquales, Quare æquales quoque erunt rectæ lineæ D E, D F, & reliqui

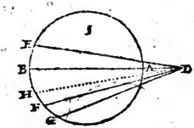
liqui anguli E D C, F D C & proinde B D E, B D F

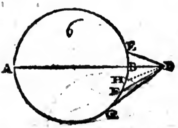
Sed ſi æquales arcus B E, F G non fuerint vtrinque à diametro, Dico angulum F D G angulo B D E minorem





eſſe, nam tunc vel arcus, qui ſunt vtrinqne à diametro B E, B F æquales fuerint, vt in 3. & 4. vt tunc quoque ęquales erunt arcus B F, F G, itemque anguli B D E, B D F, Quare ex demonſtratis minor erit angulus F D G quam B D F hoc eſt B D E; At ſi arcus, qui ſunt vtrinque à diametro B E, B F æquales non fuerint, fueritque arcus B E minor quam B F, vt in 5. & 6. tunc fiat arcus B H ęqualis arcui B E, & tunc



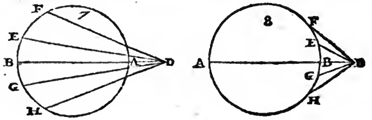


quoque æquales erunt arcus B H, F G, itemque anguli B D F, B D H, Quare ex demonſtratis minor erit angulus F D G quam B D H hoc eſt B D E

Pro expeditione ſecundæ partis, quando videlicet diameter non connumeratur inter pręfatas rectas lineas, ſi æquales arcus E F, G H ita conſtituti ſint, quod arcus B E,

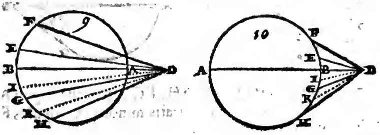
B G,

B G,qui ſunt vtrinque à diametro,inter ſe quoque ęquales fuerint,vt in 7. & 8. Dico angulos E D F, G D H æquales



eſſe,nam tunc ex demonſtratis ęquales erunt anguli B D E,B D G,ergo ſi ęqualibus arcubus B E, B G ęquales addā-tur arcus E F,G H, æquales quoque fient arcus B F,B H, & anguli B D F,B D H, Quare ſi ab ęqualibus angulis B D F, B D H ęquales auferantur anguli B D E, B D G, ęquales remanebunt anguli E D F,G D H

Sed ſi æquales arcus E F, G H ita conſtituti ſint, quod arcus B E, B G, qui ſunt vtrinque à diametro, ęquales non fuerint, fueritque arcus B E minor quam B G, vt in vltimis duabus figuris, Dico angulum G D H angulo E D F minorem eſſe, nam tunc ſi fiant arcus B I, I K



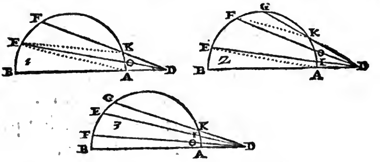
æquales arcubus B E, E F, ęquales quoque erunt arcus I K, G H, itemque anguli E D F, I D K, Quare ex demon-

monſtratis angulus G D H minor erit quam I D K hoc eſt E D F, quod erat demonſtrandum.

THEOREMA V.

SI à puncto diametri extra circulum productæ in circulum plures cadant rectæ lineæ, tamen ad eaſdem partes, cauam & conuexam circuli periferiam ſecantes, ſi æquales arcus cauos abſcinderint, erunt arcus conuexi oppoſiti inæquales, minorque ſemper erit ille, qui diametro proximior eſt.

Eſto circulus, cuius diameter B A, punctum aſſumptum D, ſint primo æquales arcus caui B E, E F continui, vt in



1. figura, Dico arcum conuexum A O arcu conuexo K O minorem eſſe, conſtat ex 1. figura ſecundi theor. quando arcus B E, E F æquales fuerint, angulum B D E angulo E D F maiorem eſſe, ductis igitur rectis lineis E K, E A æquales erunt anguli E K F, E A B, & proinde reliqui deinceps E K D, E A D, Quare angulus A E D angulo K E D,

hoc eſt arcus A O arcu K O minor erit.

Sint ſecundo æquales arcus caui B E, F G non continui, vt in 2. figura, Dico arcum conuexum A I arcu conuexo K O minorem eſſe, conſtat ex 3. figura ſecundi theor. quã-do arcus B E, F G æquales fuerint, angulum B D E angulo F D G maiorem eſſe, ductis igitur rectis lineis F K, E A, æquales erunt anguli F K G, E A B, & proinde reliqui deinceps F K D, E A D, Quare angulus A E D angulo K F D, hoc eſt arcus A I arcu K O minor erit.

Sint tertio æquales arcus caui B E, F G intercepti, vt in 3. figura, Dico arcum conuexum A I arcu conuexo K O minorem eſſe, conſtat ex 5. figura ſecundi theor. quando arcus B E, F G æquales fuerint, arcus B F, E G quoq; æquales eſſe, ergo ex antecedente caſu arcus A O minor eſt quã K I, Quare ſi inæqualibus arcubus A O, K I addatur communis arcus I O, arcus A I arcu K O minor erit, quod erat demonſtrandum.

THEOREMA VI.

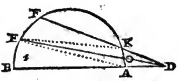
ET contra ſi præfatæ rectæ lineæ arcus conuexos æquales abſcinderint, caui oppoſiti inæquales erunt, minorque ſemper erit ille, qui à diametro longius abeſt.

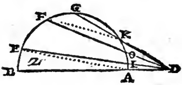
Sint primo æquales arcus conuexi A O, K O continui, vt in 1. figura, Dico arcum cauum E F arcu cauo B E minorem eſſe, conſtat ex 2. figura ſecundi theor. quando arcus A O, K O æquales fuerint, angulum A D E angulo K D E maiorem eſſe, ductis igitur rectis lineis E K, E A, æquales erunt anguli K E D, A E D, & proinde reliquus E K D reliquo E A D maior, Quare angulus E K F angulo E A B, hoc

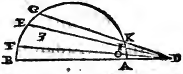
eſt

eſt arcus E F arcu B E minor erit.

Sint ſecundo æquales arcus conuexi A I, K O non con-







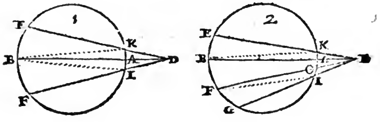
tinui,vt in 2.figura,Dico arcum cauum F G arcu cauo B E minorem eſſe,conſtat ex 4. figura ſecundi theor.quando arcus A I,K O æquales fuerunt,angulum A D E angulo F D G maiorem eſſe,ductis igitur rectis lineis F K,E A,æquales erunt anguli K F D, A E D, & proinde reliquus F K D reliquo E A D maior, Quare angulus F K G angulo E A B,hoc eſt arcus F G arcu B E minor erit.

Sint tertio æquales arcus conuexi A I, K O intercepti , vt in 3.figura,Dico arcum cauum F G arcu cauo B E minorem eſſe , conſtat ex 6. figura ſecundi theor. quando arcus A I, K O æquales fuerint , arcus A O , K I quoque æquales eſſe , ergo ex antecedente caſu arcus E G minor eſt quam B F , Quare ſi inæqualibus arcubus E G, B F addatur communis arcus F E , arcus F G arcu B E minor erit , quod erat demonſtrandum .

THEOREMA VII.

SI à puncto diametri extra circulum productæ in circulum plures cadant rectæ lineæ, tamen ad diuerſas partes, cauam & conuexam circuli periferiam ſecantes, ſi æquales arcus cauos abſcinderint, erunt arcus conuexi oppoſiti inæquales, minorque ſemper erit ille, qui diametro proximior eſt, vnico tamen excepto caſu, quando videlicet illę rectæ lineæ æquales arcus cauos abſcinderint à diametro, nam tunc æquales quoque erunt & conuexi.

Pręſens theorema bipartitum eſt, nam vel inter has rectas lineas connumeratur diameter, vel nō; Pro expeditione primę partis, eſto circulus, cuius diameter B A, punctum aſſumptum D, rectę lineę D E, D F, quę cum diametro producta B D vtrinque abſcindant ęquales arcus cauos B E,

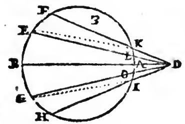


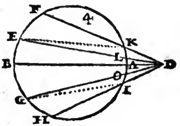
B F, vt in 1. figura, Dico arcus conuexos K A, I A quoque ęqua-

ęquales eſſe, conſtat ex 1. figura quarti theor. quando arcus B E, B F ęquales fuerint, angulos B D E, B D F quoque æquales eſſe, ductis igitur rectis lineis B K, B I, ęquales erũt anguli B K E, B I F, & proinde reliqui deinceps B K D, B I D, Quare anguli K B D, I B D, hoc eſt arcus K A, I A inter ſe quoque ęquales erunt.

Sed ſi ęquales arcus caui B E, F G non fuerint vtrinque à diametro, vt in 2. figura, Dico arcum conuexum K A arcu conuexo I O minorem eſſe, conſtat ex 3. & 5. figura quarti theor. quando arcus B E, F G ęquales fuerint, angulum B D E angulo F D G maiorem eſſe, ductis igitur rectis lineis B K, F I, ęquales erunt anguli B K E, F I G, & proinde reliqui deinceps B K D, F I D, Quare angulus K B D, angulo I F D, hoc eſt arcus K A arcu I O minor erit.

Pro expeditione ſecundæ partis, quando videlicet diameter non connumeratur inter prędfatas rectas lineas, ſi ęquales arcus caui E F, G H ita conſtituti ſint, quod arcus B E, B G, qui ſunt vtrinque à diametro, inter ſe quoque ęquales fuerint, vt in 3. figura, Dico arcus conuexos K L,





I O ęquales eſſe, conſtat ex 7. figura quarti theor. quando arcus E F, G H æquales fuerint, angulos E D F, G D H quoque ęquales eſſe, ductis igitur rectis lineis E K, G I, ęquales erunt anguli E K F, G I H, & proinde reliqui deinceps E K D, G I D, Quare anguli K E D, I G D, hoc eſt arcus K L, I O

inter

inter ſe quoque æquales erunt,

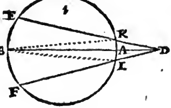
Sed ſi ęquales arcus caui E F, G H ita conſtituti ſint; quod arcus B E, B G, qui ſunt vtrinque à diametro, ęquales non fuerint, fueritque arcus B E minor quam B G, vt in 4. figura, Dico arcum conuexum K L arcu conuexo I O minorem eſſe; conſtat ex 9. figura quarti theor. quando arcus E F, G H ęquales fuerint, angulum E D F angulo G D H maiorem eſſe, ductis igitur rectis lineis E K, G I, ęquales erunt anguli E K F, G I K, & proinde reliqui deinceps E K D, G I D, Quare angulus K E D angulo I G D, hoc eſt arcus K L arcu I O minor erit, quod erat demonſtrandum.

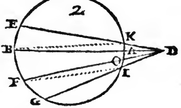
THEOREMA VIII.

ET contra ſi præfatæ rectæ lineæ arcus conuexos æquales abſcinderint, caui oppoſiti inæquales erunt, minorque ſemper erit ille, qui à diametro longius abeſt, vnico tamen excepto caſu, quando videlicet illę rectæ lineæ æquales arcus conuexos abſcinderint à diametro, nam tunc æquales quoque erunt & caui.

Pręſens theorema bipartitum eſt, nam vel inter has rectas lineas connumeratur diameter, vel non; Pro expeditione primę partis, eſto circulus, cuius diameter B A, punctum aſſumptum D, rectæ lineæ D E, D F, quæ cum diametro producta B D vtrinque abſcindant æquales arcus conuexos K A, I A, vt in 1. figura, Dico arcus cauos B E, B F quoque æquales eſſe, conſtat ex 2. figura quarti theor. quan-

quando arcus K A, I A æquales fuerint, angulos B D E, B D F quoque æquales eſſe, ductis igitur rectis lineis B K, B I,

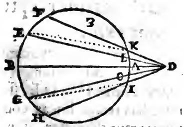


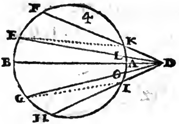


æquales erũt anguli K B D, I B D, & proinde reliquĩ ęquales B K D, B I D, Quare anguli deinceps B K E, B I F, hoc eſt arcus B E, B F inter ſe quoque ęquales erunt .

Sed ſi æquales arcus conuexi K A, I O non fuerint vtrinque à diametro, vt in 2. figura, Dico arcum cauum F G arcu cauo B E minorem eſſe, conſtat ex 4. & 6. figura quarti theorema quando arcus K A, I O ęquales fuerint, angulum B D E angulo F D G maiorem eſſe, ductis igitur rectis lineis B K, F I, æquales erunt anguli K B D, I F D, ſed F I D maior quam B K D; Quare angulus F I G angulo B K E, hoc eſt arcus F G arcu B E minor erit.

Pro expeditione ſecundæ partis, quando videlicet dia-





meter non connumeratur inter præfatas rectas lineas, ſi

ęqua-

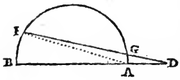
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æquales arcus conuexi K L,I O ita conſtituti ſint, quod arcus A L,A O, qui ſunt vtrinque à diametro, inter ſe quoq; æquales fuerint,vt in 3.figura, Dico arcus cauos E F, G H æquales eſſe, conſtat ex 8. figura quarti theor. quando arcus K L,I O ęquales fuerint,angulos E D F, G D H quoque æquales eſſe,ductis igitur rectis lineis E K,G I,ęquales erunt anguli K E D,I G D, Quare reliqui E K D, G I D, & proinde anguli deinceps E K F, G I H, hoc eſt arcus E F, G H inter ſe quoque æquales erunt.

Sed ſi ęquales arcus conuexi K L, I O ita conſtituti ſint, quod arcus A L,A O, qui ſunt vtrinque à diametro, ęquales non fuerint,fueritque arcus A L minor quam A O,vt in 4.figura,Dico arcum cauum G H arcu cauo E F minorem eſſe, conſtat ex 10. figura quarti theor.quando arcus K L,I O ęquales fuerint, angulum E D F angulo G D H maiorem eſſe,ductis igitur rectis lineis E K, G I, ęquales erũt anguli K E D,I G D,ſed G I D maior quam E K D, Quare angulus G I H angulo E K F,hoc eſt arcus G H arcu E F minor erit,quod erat demonſtrandum.

THEOREMA IX.

SI à puncto diametri extra circulũ productę in cauam circuli periferiam duę cadant rectæ lineæ, arcus cauus interceptus arcu conuexo ſemper maior erit.



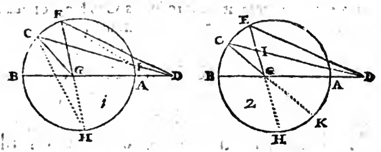
Eſto circulus,cuius diameter B A,punctum aſſumptum D,rectę lineę D B, D E, ſiue inter has rectas lineas connumeretur diameter, ſiue non, Dico arcum cauum B E arcu

arcu conuexo A G maiorem eſſe, nam ducta recta A E, erit angulus B A E maior quam angulus A E G, & proinde arcus B E maior quam A G, quod erat demonſtrandum.

THEOREMA X.

SI ad duo puncta diametri, quorum vnū fuerit intra circulum, alterum extra, duo conſtituantur anguli eidem arcui cauo inſiſtentes, angulus intra circulum exteriore ſemper maior erit.

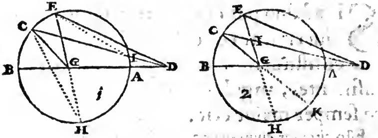
Eſto circulus, cuius diameter B A, puncta aſſumpta G, D, arcus C E, Dico angulum C G E angulo C D E maiorem



eſſe, nam ſi recta E G protrahatur ad H, ducanturque rectę E I, C H, vt in 1. figura, erunt anguli C H E, C I E inter ſe ęquales, ſed C G E maior eſt quam C H E, hoc eſt quam C I E, & hic maior quam C D E, Quare angulus C G E angulo C D E multo maior erit, quod erat demonſtrandum.

ALITER:

POteſt & aliter demonſtrari pręſens theorema, nam ſi rectę E G, C G protrahantur ad H & K, vt in 2. figura, erit angulus D E H maior quam D C K, Quare ſi conſideremus duo triangula I C G, I D E, cum anguli aduerticem I ęquales inter ſe fuerint, erit reliquus C G E maior quam reliquus C D E, quod erat demonſtrandum.

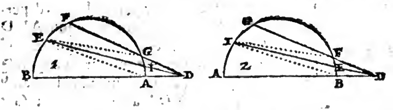


THEOREMA XI.

SI à puncto diametri extra circulum productæ in cauam vel conuexam circuli periferiam plures cadant rectæ lineę, tamen ad eaſdem partes, arcus diametro proximior ad ſuum viciniorem ſemper minorem habebit rationem, quam angulus ad angulum.

Eſto circulus, cuius diameter B A, punctum aſſumptum D, Dico arcum cauum B E ad arcum cauum E F minorem habere rationem, quam angulus B D E ad angulum E D F, pro-

protrahantur rectę E G, E A, vt in 1. figura, & erunt conſti-tuta duo triangula E A D, E G D ſuper communi baſe E



D, cum itaque in triangulo E A D baſis E D diuidatur in I, ſitque ſegmentum D I maius latere contermino D A, erit ex Lemmate quodam Apollonij, quo vtitur Ptol. lib. 12. Almageſti cap. 1. maior proportio D I ad I E quam anguli D E A ad angulum A D E, & conuertendo maior proportio anguli A D E ad angulum D E A quam I E ad D I, at in reliquo triangulo E G D baſis D E diuiditur in I, eſtq; ſegmentum I E maius latere contermino E G, ergo eadem ratione maior erit proportio I E ad D I quam anguli E D G ad angulum G E D, ex æquo igitur maior erit proportio anguli A D E ad angulum D E A quam anguli E D G ad angulum G E D, & conuertendo & componendo minor erit proportio angulorum D E A, A D E, hoc eſt anguli E A B ad angulum B D E quam angulorum G E D, E D G, hoc eſt anguli F G E ad angulum E D F, & permutando minor erit proportio anguli E A B ad angulum F G E, hoc eſt arcus caui B E ad arcum cauum E F quam anguli B D E ad angulum E D F

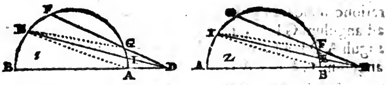
Dico etiam arcum conuexum A I ad arcum conuexum I G minorem habere rationem quã angulus A D I ad angulum I D G, demonſtratum enim eſt quod arcus cauus B E ad arcum cauum E F minorem habet rationem quam angulus B D E ad angulum E D F, ſed arcus B E, E F ſe habent vt anguli E A B, F G E, eſtque angulus E A B æqualis

D 2 duo-

duobus angulis A E D, A D E, itemque angulus F G E duobus G E D, G D E, quare minor erit proportio duorum angulorum A E D, A D E ſimul ad duos angulos G E D, G D E ſimul quam anguli B D E ad angulum E D F, & permutando & diuidendo minor erit proportio anguli A E D ad angulum A D I quam anguli G E D ad angulum I D G, iterumque permutando minor erit proportio anguli A E D ad angulum G E D, hoc eſt arcus conuexi A I ad arcũ conuexum I G quam anguli A D I ad angulum I D G, quod erat demonſtrandum.

ALITER.

Poteſt & aliter demõſtrari prȩſens theorema, exordiẽdo ab arcubus conuexis, vt in 2. figura, nam cũ D E ad I E maiorem habent rationem quam angulus B I D ad angulũ



B D I, erit conuertendo maior proportio anguli B D I ad angulum B I D, quam I E ad D E, ſed I E ad D E maiorem habet rationem quam angulus F D I ad angulum F I D, ex æquo igitur maior erit proportio anguli B D I ad angulum B I D quam anguli F D I ad angulum F I D, & conuertendo & permutando erit minor proportio anguli B I D ad angulum F I D hoc eſt arcus conuexi B E ad arcum conuexum E F quam anguli B D E ad angulum E D F.

Dico etiam arcum cauum A I ad arcum cauum I G minorem habere rationem quam angulus A D I ad angulum I D G,

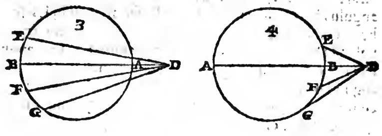
IDG, demonſtratum enim eſt quod arcus conuexus BE ad arcum conuexum EF minorem habet rationem quam angulus BDE ad angulum EDF, ſed arcus BE, EF ſe habent vt anguli BID, FID, minor igitur erit proportio anguli BID ad angulum FID quam anguli BDI ad angulũ FDI, & permutando & componendo minor erit proportio anguli IBA ad angulum IFG, hoc eſt arcus caui AI ad arcum cauum IG quam anguli ADI ad angulum IDG, quod erat demonſtrandum.

THEOREMA XII.

SI à puncto diametri extra circulũ productæ in cauam vel conuexam circuli periferiam plures cadant rectę lineæ, tamen ad diuerſas partes vtrinque à diametro, arcus diametro proximior ad reliquum ſemper minorem habebit rationem quam angulus ad angulum, vnico tamen excepto caſu, quando arcus vtrinque à diametro æquales fuerint, nam tunc eandem rationem habebunt & arcus & anguli.

Eſto circulus, cuius diameter BA, punctum aſſumptum D, rectæ lineæ DE, DF, quæ cum diametro producta DB vtrinque abſcindant æquales arcus BE, BF, Dico arcum BE ad arcum BF eandem habere rationem quam angulus BDE ad angulum BDF, nam tunc æquales quoque erunt anguli BDE, BDF, Quare cum inter arcus & angulos reperiatur proportio æqualitatis, arcus eandem habebunt ratio-

rationem quam anguli.



Sed ſi ſumantur duo arcus inæquales B E, B G, fueritque arcus B E diametro proximior minor quam B G, Dico arcum B E ad arcum B G minorem habere rationem quam angulus B D E ad angulum B D G, nam tunc ſi fiat arcus B F æqualis arcui B E, æqualis quoque erit angulus B D F angulo B D E, At ex demonſtratis minor eſt proportio arcus B F ad arcum F G quam anguli B D F ad angulum F D G, quare componendo & per conuerſionem rationis maior erit proportio arcus B G ad arcum B F quam anguli B D G ad angulum B D F, & conuertendo minor erit proportio arcus B F hoc eſt B E ad arcum B G quam anguli B D F hoc eſt B D E ad angulum B D G, quod erat demonſtrandum.

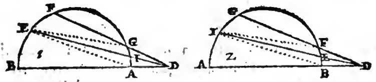
THEOREMA XIII.

SI à puncto diametri extra circulum productæ in circulum plures cadant rectæ lineæ, tamen ad eaſdem partes, cauam & conuexam circuli periferiam ſecantes, ar-

cus

cus cauus diametro proximior ad ſuũ cauũ viciniorem ſemper maiorem habebit rationem quam arcus conuexus ad conuexum.

Reaſſumpta figura 1. theor. 11. Dico arcum cauum B E ad arcum cauum E F maiorem habere rationem quam arcus conuexus A I ad arcum conuexum I G, demonſtratum

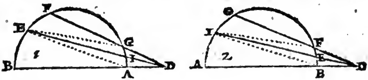


eſt enim in eo theoremate, quod minor eſt proportio duorum angulorum A E D, A D E ſimul, hoc eſt anguli E A B ad angulum A D E, quam duorum angulorum G E D, G D E ſimul, hoc eſt anguli F G E ad angulum G D E, quare per conuerſionem rationis & permutando maior erit proportio anguli E A B ad angulum F G E, quam anguli A E D ad angulum G E D, ſed anguli E A B, F G E ſe habent vt arcus caui B E, E F, itemque anguli A E D, G E D vt arcus conuexi A I, I G, maior ergo erit proportio arcus caui B E ad arcum cauum E F quam arcus conuexi A I ad arcum cõuexum I G, quod erat demonſtrandum.

ALITER.

Poteſt & aliter demonſtrari præſens theorema exordiẽdo ab arcubus conuexis, reaſſumpta figura 2. theor. 11. Dico arcum cõuexum B E ad conuexnm E F minorem habere ratiouem quã cauus A I ad cauum I G, demonſtratũ eſt enim in eo theoremate quod maior eſt proportio anguli

guli B D I ad angulum B I D quam anguli F D I ad angulum F I D, quare componendo maior erit proportio duo-



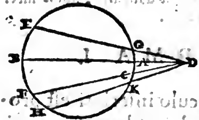
rum angulorum B D I, B I D ſimul, hoc eſt anguli I B A ad angulum B I D, quam duorum angulorum F D I, F I D ſimul, hoc eſt anguli G F I ad angulum F I D, & conuertendo & permutando minor erit proportio anguli B I D ad angulum F I D quam anguli I B A ad angulum G F I, ſed anguli B I D, F I D ſe habent vt arcus conuexi B E, E F, itemque anguli I B A, G F I vt arcus caui A I, I G, minor ergo erit proportio arcus conuexi B E ad arcum conuexum E F quam caui A I ad cauum I G, quod erat demonſtrandum.

THEOREMA XIV.

SI à puncto diametri extra circulum producta in circulum plures cadant rectæ lineæ, tamen ad diuerſas partes vtrinque à diametro, cauam & conuexam circuli periferiam ſecantes, arcus cauus diametro proximior ad reliquum cauum ſemper maiorem habebit rationem quam arcus conuexus ad conuexum, vnico tamen excepto caſu,

ſu , quando arcus caui vtrinque à diametro æquales fuerint, nam tunc eandem rationem habebunt & caui & conuexi.

Eſto circulus, cuius diameter BA, punctum aſſumptum D, rectæ lineæ D E, D F, quæ cum diametro producta D B vtrinque abſcindant æquales arcus cauos BE, B F, Dico arcum cauum B E ad cauum B F eandem habere rationem, quam arcus conuexus A G ad conuexum A C, nam tunc æquales quoque erunt & conuexi, Quare cum inter arcus cauos & conuexos reperiatur proportio æqualitatis, eandem rationem habebunt caui quam conuexi.



Sed ſi ſumantur duo arcus caui inæquales B E, B H, fueritque B E diametro proximior minor quam BH, Dico arcum cauum BE ad cauum BH maiorem habere rationem, quam arcus conuexus AG ad conuexum AK, nam tunc ſi fiat arcus cauus B F æqualis cauo B E, æqualis quoque erit arcus conuexus C A conuexo AG, At ex demonſtratis maior eſt proportio arcus caui BF ad cauum F H quam arcus conuexi CA ad conuexum CK, quare componendo & per conuerſionem rationis minor erit proportio arcus caui B H ad cauum BF quam arcus conuexi AK ad conuexum CA, & conuertendo maior erit proportio arcus caui B F hoc eſt caui B E ad cauum B H, quam arcus conuexi C A hoc eſt conuexi A G ad conuexum AK, quod erat demonſtrandum.

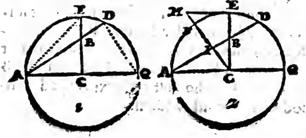
EXERCITATIO SECVNDA.

IN hac exercitatione proponuntur quædam theoremata ab autoribus abſque demonſtratione ad nos transmiſſa, & quædam alia à nobis inter legendum animaduerſa.

THEOREMA I.

LAtus quadrati circulo inſcripti eſt proportionale inter hypothenuſam periferiæ, & ſinum duplum reſiduæ.

Hoc eſt conſectarium inſigne apud Vietam in libro illo ſingulari ad canonē Mathematicum, in fine cap. 13. quod facillimè demonſtrari poteſt; eſto circulus, cuius diameter A G, & ex centro C erigatur perpendicularis C E, à punctoque A ducantur rectę A E, A D, & A D ſecet C E in B, erit AE latus quadrati circulo inſcripti, A B hypothenuſa pe-



ciſe-

riferiæ,& A D ſinus duplus reſiduæ; ſed fortaſsis aliquibus obſcurum videbitur, quomodo A B ſit hypothenuſa periferiæ,& A D ſinus duplus reſiduæ, erunt iſthæc ſanè declaranda, aſſumatur 2. figura, in eaque fiat triangulum rectangulum C E H, recta C H dicitur à Vieta hypothenuſa periferiæ ſiue arcus E F, arcus reliquus F A dicitur periferia reſidua, ex A in C F demittatur normalis A I, ad circumferentiam in D producta,& erit A I ſinus reſiduæ F A, ſed quia A D diuiditur bifariam in I, erit A D ſinus duplus reſiduæ F A, Quod autem A B ſit æqualis hypothenuſæ C H, ex eo conſtat, quia cum E H parallela ſit ad C A, angulus E H C æquabitur angulo A C I, ſunt & anguli C E H, C I A recti, ergo angulus C A B æqualis erit angulo E C H, quare triãgula C E H, B C A ſimilia erunt, igitur vt C A ad A B, ſic E C ad C H,& permutando vt C A ad E C, ſic A B ad C H, ac æquales ſunt C A, E C, quia ſemidiametri, ergo A B æqualis quoque erit hypothenuſæ C H.

Redeundo itaque ad primam figuram, Dico quod A E proportionalis eſt inter A B & A D, nam ducta recta D G, ſimilia erunt triangula A B C, A D G, ergo vt A C ad A B, ſic A D ad A G, igitur planum C A G ęquabitur plano B A D, ſed plano C A G ęquatur duplum quadratum ex A C, hoc eſt quadratum ex A E, ergo quadratum ex A E plano B A D æquale erit, quare A E proportionalis eſt inter A B & A D, quod erat oſtendendum.

SCHOLIVM.

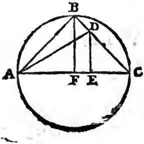
PRæſens theorema proponi poteſt ſecundum terminos cõmunes, hoc modo, latus quadrati circulo inſcripti eſt proportionale inter ſecantem arcus & ſinum duplum complementi, vel generalius, quadrantis ſubtenſa eſt proportionalis inter ſubtenſam ab eodem termino ductam at-

que eius portionem inter communem terminum & perpendicularem à centro ad terminum ſuperiorem quadrantis interceptam, vt in prima figura, recta A E proportionalis eſt inter ſubtenſam A D & eius portionem A B, itemque inter diametrum A G & ſemidiametrum A C, nec non inter omnes ſubtenſas ductas à puncto A in quadrantem oppoſitum E G & earum portiones inter A & perpendicularem C E interceptas; Hinc deducitur facilis praxis, quomodo ex data ſubtenſa cognoſcere poſſimus ſecantē arcus, ſeu vt loquitur Vieta hypothenuſam periferię, vt in 2. figura, ſi data ſubtenſa A D bifariam ſecetur in I, erit A I ſinus arcus A F, & portio intercepta A B erit ſecans arcus reliqui F E, inſuper C B erit tangens eiuſdem arcus F E, æqualis eſt enim C B ipſi E H.

THEOREMA II.

Latus quadrati circulo inſcripti conſtat exceſſu, quo à diametro ſuperatur, & inſuper potente duplum exceſſum.

Hoc theorema indemonſtratum ponitur à Vieta in eodem libro ad canonem Mathematicum, cap. 25. num. 12. quod à nobis ita demonſtratur; Eſto circulus, cuius diameter A C, & à cētro F erigatur normalis F B, ductęque A B, C B erunt latera quadrati circulo inſcripti, ex A C abſcindatur A E ipſi A B ęqualis, & ponatur E D ad F B parallela, connectaturque A D; Apparet manifeſto ſimilia eſſe triangula A B C, D E C,



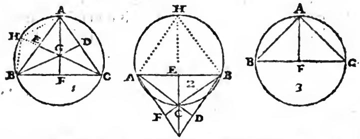
E C,& obid rectas D E,E C æquales eſſe.ſed E C eſt exceſſus diametri A C ſuper A E,hoc eſt A B latus quadrati, ergo D C poteſt duplum exceſſum ; item duo triangula A B D,A D E rectangula ſunt, igitur quadrata A B, B D ęqualia erunt quadratis A E,E D,Quare ablatis ęqualibus quadratis A B,A E,ęqualia remanebunt quadrata B D,E D,& cõſequenter ęquales erunt rectę B D, E D, ſed E D ęquatur exceſſui E C,ergo B C latus quadrati circulo inſcripti conſtat ex B D exceſſu, quo à diametro ſuperatur, & ex D C potente duplum exceſſum,quod erat oſtendendum.

THEOREMA III.

SI ab angulis cuiuſcunque trianguli in oppoſita latera perpendiculares demittantur, omnes in eodem puncto ſeſe interſecabunt.

Regiomontanus lib. primo de triangulis Prop. 32. dicit ſe alibi hoc demonſtraſſe,in quo verò loco id fecerit, prorſus ignoro, tamen ego aſſumptum theorema ſic demõſtrarem;Proponatur triangulum A B C,vt in 1.figura, acutangulum, in 2.obtuſangulum,& angulus obtuſus ſit ad C, ab angulo B in latus oppoſitum A C demittatur normalis B D,ab angulo C in latus oppoſitum A B normalis C E, mutuo ſe ſecantes in G, demum ab angulo A deſcendat in latus oppoſitum B C recta A F,tranſiens per punctum G,Dico rectam A F ad latus B C perpendicularem eſſe ; circa triangulum A B C deſcribatur circulus A H B C , & normalis C E protracta ſecet periferiam in H, ducanturque rectę B H,A H ; Manifeſtum eſt in primis triangula E G B, D G C eſſe ęquiangula, ideoq; ęquales eſſe angulos D C G, E B G,

E B G, hoc eſt A C H, A B G, angulo vero A C H ęquatur angulus H B A, inſiſtunt enim eidem arcui H A, quare an-



guli H B A, A B G quoque inter ſe æquales erunt, quaprop ter in triangulo H B G æquales quoque erunt anguli B H G, B G H, baſiſque H G bifariam ſecabitur in E, ideoque in altero triangulo H A G anguli A H G, A G H inuicem æquabuntur: Cum itaque angulus B G H ęqualis ſit angulo B H G, itemque angulus A G H angulo A H G, erit totus angulus B G A hoc eſt F G D toti angulo B H A ęqualis.

Quamobrem cum quadrilaterum B H A C ſit in circulo, erunt duo anguli oppoſiti B H A, B C A duobus rectis ęquales, oſtenſus eſt angulus F G D ęqualis angulo B H A, ergo in quadrilatero D G F C duo anguli oppoſiti F G D, D C F duobus rectis ęquales quoque erunt, At cuiuslibet quadrilateri quatuor anguli ſimul ſumpti quatuor rectis adæquantur, cum itaque oſtenſum ſit duos oppoſitos angulos F G D, D C F duobus rectis adæquari, quare reliqui duo oppoſiti G D C, C F G ęquales erunt duobus rectis, eſtque G D C rectus per fabricam, ergo reliquus angulus C F G rectus quoquē erit, & conſequenter recta A F tranſiens per punctum G, perpendicularis erit ad latus oppoſitum B C

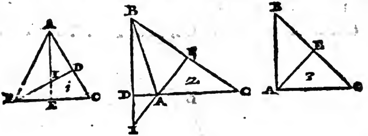
In triangulo rectangulo hoc ſatis manifeſtum eſt, nulla ſanè indigemus demonſtratione, vt in 3. figura, normalis enim

enim quæ ducitur ab angulo B in latus oppoſitum A C, idē eſt quod latus BA, normalis quæ ducitur ab angulo C in latus oppoſitum A B, idem eſt quod latus C A, quare ſi ab angulo A in latus oppoſitum B C demittatur normalis AF, hæc cum duabus reliquis in eodem puncto A ſe interſecabunt, verū eſt igitur in omnibus triangulis perpendiculares ab angulis ductas in oppoſita latera, in vno, & eodem puncto ſeſe interſecare, quod erat oſtendendum.

THEOREMA IV.

SI ab vno æqualium angulorum iſoſcelis trianguli in latus oppoſitum perpendicularis demittatur, faciet hæc cum baſi angulum æqualem dimidio anguli verticalis.

Proponatur triangulum iſoſceles A B C, vt in 1. figura, acutangulum, in 2. obtuſangulum, cuius æqualia latera ſint A B, A C, & ab angulo B in latus oppoſitum A C



demittatur perpendicularis BD; Dico angulum B C D eſſe dimidium anguli B A C, ab angulo A cadat normalis A E

in

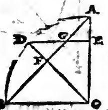
in latus oppoſitum B C, ſeçans AE in I, & angulus B A C biſariam ſectus erit; Cum itaq; in duobus triangulis AID, BIE anguli ad E & D æquales ſint, nempè recti, æquales ſunt quoque anguli ad I, ergo reliqui duo IAD, IBE, hoc eſt EAC, DBC inter ſe quoque æquales erunt, eſt autem angulus EAC dimidium anguli B A C, quare angulus DBC eiuſdem BAC dimidium erit.

In triangulo iſoſcele rectangulo hoc ſatis manifeſtum eſt, vt in 3. figura, normalis enim BA, quæ ducitur ab angulo B in latus oppoſitum AC, idem eſt quod latus BA, & anguli ABE, BAE inuicem adæquantur, ſunt enim ambo ſemirecti, quod erat oſtendendum.

THEOREMA V.

SI circuli quadrans biſariam ſectus fuerit, & à puncto ſectionis in diametrum deſcendat normalis alteri diametro parallela, ſecabit hæc chordam quadrantis in duas partes inæquales, quarum maior ſemidiametro æqualis erit.

Eſto circuli quadrans ACB, cuius arcus A B biſariam



diuidatur in D, à quo deſcendat normalis D E, ſecans chor-

dam quadrantis in G, Dico G B ſemidiametro æqualem eſſe; Nam ducta ſemidiametro C D ſecans chordam in F, conſtituentur tria triangula iſoſcelia DEC, BFC, DFG prioris duo latera D E, C E æqualia ſunt, nam anguli ad C & D ęquales ſunt, nempe ſemirecti, ſecundi duo latera C F, B F, quia anguli ad C & B ſemirecti, demum tertij duo latera FD, FG, quia anguli ad D & G ſemirecti, Quare ſi æqualibus lateribus B F, C F addantur æqualia latera F G, F D, æquales conſtituentur rectæ B G, C D, nempe maior portio chordæ adæquabitur ſemidiametro, quod erat oſtendendum.

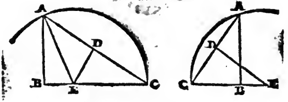
EXERCITATIO TERTIA.

IN hac Exercitatione deſcribuntur problemata quædam, partim mihi ad ſoluendum propoſita, partim inter legendum à nobis animaduerſa.

PROBLEMA I.

DAto triangulo rectangulo, in eius baſe etiam producta punctum inuenire, à quo ſi circulus deſcribatur tranſeat is per extremitates hypothenuſæ.

Eſto triangulum rectangulum ABC, eius baſis B C hypothenuſa AC bifariam diuidatur in D, ex quo puncto ad ipſam eandem excitetur normalis, occurrens baſi in E, Di-



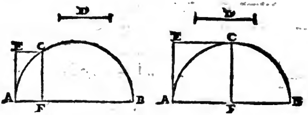
co E punctum eſſe quæſitum, nam ducta EA, erunt duo latera

tera A D, D E æqualia duobus lateribus C D, D E anguloſque continent ęquales, nempe rectos, ergo & baſes E C, E A inter ſe quoque æquales erunt, quare ſi centro E & interuallo E A circulus deſcribatur, neceſſario tranſibit is per punctum C; factumque erit quod fieri oportebat.

PROBLEMA II.

IN dati circuli periferia punctum inuenire, à quo cum in diametrum perpendicularis cadat, ea datæ rectæ lineæ ſit ęqualis, oportet datam rectam lineam ſemidiametro non eſſe maiorem.

Eſto ſemicirculus ACB, data recta linea D, à puncto A ſuper diametrum AB excitetur perpendicularis AE, ęqua-

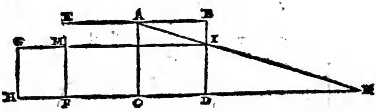


lis datæ rectæ lineę D, & ab E ducatur E C ipſi diametro parallela, occurrens periferiæ in C, Dico C punctum eſſe quæſitum, à puncto itaque C cadat in diametrum normalis C F, quare cum parallelogrammum ſit A C, erit recta C F rectę E A, hoc eſt datæ D æqualis, adinuenimus ergo punctum in periferia, quod quærebatur.

PROBLEMA III.

AB angulo dati quadrati lineam rectam ducere, quę ſecet latus oppoſitum, & concurrat cum reliquo latere inferiori producto, ita vt triangulum exterius factum dato quadrato ſit æquale.

Hoc problema demonſtratum adinuenimus à Nicolao Tartalea per longas ambages, itemque ab Alexandro Anderſono tamen generalius & elegantius, ſed profecto hoc etiam modo demonſtrari poteſt; Datum quadratum eſto A D, & ab angulo A ducenda ſit recta linea, quę propoſitum abſoluat, lineę BA, DC duplētur ad E & F, compleaturque rectangulum DE, quod dati quadrati A D duplum erit, amplius ſuper D F applicetur rectangulum DG ęquale rectangulo DE, excedens quadrato GF, demum ab angulo A dati quadrati ducatur recta AI occurrens lateri C, D producto in K, Dico rectam A I K abſoluere problema, triangulumque externum D I K dato quadrato A D ęquale eſſe; Nam cum rectangulum DG æquale factum ſit ipſi D E, ablato communi D M, remanet rectangulum B M æquale

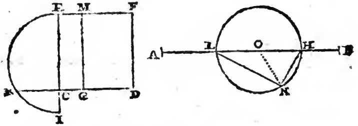


quale quadrato G F, ergo vt I M ad M G, hoc eſt D F ad DI, ſic FM ad ME, hoc eſt DI ad IB, ſed ob ſimilitudinem triangulorum DIK. B I A, vt D I ad I B ita DK ad BA, ergo vt D F ad D I ita D K ad B A, rectangulumque ſub D F, B A, hoc eſt rectangulum D E rectangulo ſub D K, DI æquale erit, quare ipſorum dimidia, quadratum videlicet A D & triangulum D I K æqualia quoque inter ſe erunt, factum eſt igitur quod ab initio proponebatur.

PROBLEMA IV.

EX data recta linea triangulum efformare, cuius anguli ita diſponantur, vt vnus vnius tertia pars ſit, & alterius pars dimidia.

Data recta linea ſit AB, fiat quadratum C F, à quo abſcindatur pars tertia CM per rectam MG, & EC producatur ad I, ſintque æquales C I, CG, ſuperque E I deſcripto



ſemicirculo, ad eius periferiam in K extendatur DC, fecetur poſtea recta propoſita in L vt A L ad L B ſit vt K C ad C D, & ex L B auferatur pars tertia B H, ſuper L H deſcripto circulo, cuius centrum O, accomodetur in eo recta

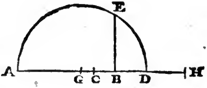
HN ipſi

H N ipſi BH æqualis, ducaturque LN, Dico triangulum L N H quęſito ſatisfacere; Nam connexa N O, erit triangulum NOH ęquilaterũ, ergo angulus ad H cõtinebit duas tertias partes vnius recti, angulus LNH rectus eſt, quia in ſemicirculo, quare angulus ad L erit tertia pars vnius recti, igitur in triangulo efformato LNH, erit angulus ad L tertia pars anguli ad N, & pars dimidia anguli ad H; Quod vero latera præfati triãguli adæquentur rectę propoſitæ A B manifeſtum eſt, ſunt enim rectę B H, HN æquales per fabricam, at AL, LN æquales per demonſtrationem, nam cũ AL ad L B ſit vt K C ad CD, proportionalia quoq; erunt quæ ſuper his deſcribuntur quadrata, eſt quadratum ex K C pars tertia quadrati ex C D, igitur quadratum ex A L quadrati ex LB tertia quoque pars erit, ſed eiuſdem quadrati ex L B pars tertia eſt quadratum ex L N, quia L N triplũ poteſt ipſius NH, at eiuſdem NH ipſa LB poteſt nonuplum, quare ęquales ſunt AL, LN, factumque eſt igitur quod fieri oportebat.

PROBLEMA V.

DAtam rectam lineam inęqualiter ſectam ita augere, vt tota compoſita ad adiunctam ſit vt maius ſegmentũ ad minus.

Recta AB ſecta inęqualiter in C producatur ad D, ſint-



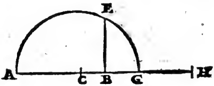
que æquales CB, BD, ſuperque A D deſcripto ſemicirculo,

lo,ad eius periferiam occurrat normalis BE, ſegmentorum A C, C B differentia eſto B G, duabuſque datis G B, B E adiungatur tertia proportionalis B H, Dico rectam BH eſſe lineam quęſitam, ac ita ſe habere A H ad B H vt AC ad CB, rectangulum enim A B D, quia ęquatur quadrato B E, æquale eſt rectangulo G B H, ergo A B ad GB erit vt BH ad B D, & permutando & componendo erit AH ad B H vt G D ad B D, hoc eſt A C ad C B, adinuenimus igitur lineam adiungendam, quod erat faciendum.

PROBLEMA VI.

Datam rectam lineam inæqualiter ſectam ita augere, vt tota compoſita ad datam ſit vt maius ſegmentum ad minus.

Recta AB ſecta ſit inæqualiter in C, ſegmentorum AC, C B differentia eſto B G, ſuperque A G deſcripto ſemicirculo, ad eius periferiam occurrat normalis BE, duabuſque



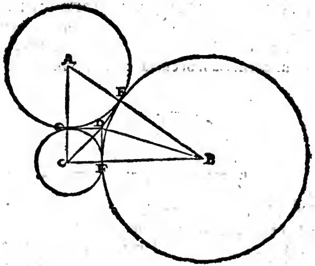
datis CB, BE adiungatur tertia proportionalis B H, Dico rectam BH eſſe lineam quæſitam, ac ita ſe habere AH ad AB, vt A C ad CB; rectangulum enim A B G ęquatur rectãgulo CBH, ergo AB ad CB erit vt BH ad BG, & permutando & componendo AH ad BH vt CG ad B G, & per conuerſionem rationis erit A H ad AB vt CG hoc eſt AC ad C B, adinuenimus igitur lineam adiungendam, quod erat faciendum.

PRO-

PROBLEMA VII.

EX datis tribus punctis tanquam centris tres circulos mutuos ſe tangentes deſcribere, oportet tria data puncta non eſſe in eadem recta linea.

Hoc quæſitum fuit mihi propoſitum à Ioanne Franciſco Sagredo patritio Veneto, meiq; amiciſſimo, Data puncta ABC inuicem connexa conſtituant triangulum, cuius duo



anguli veluti B & C biſariam diuidantur per rectas B D, C D mutuo ſe ſecantes in D, à quo puncto ad tria latera

trian-

trianguli demittantur normales D E, D F, D G, & erunt rectę B E, B F ęquales, item C F, CG, item A E, A G ex demõſtratione Prop. 4. Quarti element. Quare ſi centro B & interuallo B E fiat circulus, is tranſibit per F, centro A & interuallo A E, is tranſibit per G, centro C & interuallo C G, is tranſibit per F, & conſequenter circuli mutuo ſe contingent; quod erat faciendum.

EXERCITATIO QVARTA.

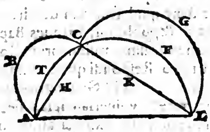
CVm eſſem Venetijs anno 1611. Georgius Fuggerus Baro in Kirchberg & VVeiſſenhorn pro Rodulpho II. tunc temporis apud Venetos Legatus quæſiuit à me quid ſentirem de circuli quadratura, quam excogitauit & publicauit Ioannes Baptiſta Porta in libello illo elementorum curuilineorum Romę impreſſo anno ſuperiori 1610. Reſpondi quadraturam illam non eſſe idoneam, petijt à me vt id Geometrice demonſtrarem, cui vt morem gererem, vniuerſam ipſam de circulo quadrando tractationem, quam exhibet autor toto libro tertio, ad quatuor tantum Propoſitiones contraximus, & confutationem noſtram ad rei propoſitę veritatem ſtabiliendam adiunximus.

Quadratura Io. Baptiſtæ Portæ.

PROPOSITIO I.

SI in ſemicirculo triangulum inſcribatur, & ſuper lateribus delineentur ſemicirculi, lunulæ ex decuſſatione ſemicirculorum efformatæ inſcripto triangulo æquales erunt.

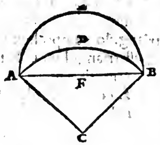
In ſemicirculo A C F L ſit inſcriptum triangulum A C L, & ſuper lateribus A C, C L delineati quoque ſint ſemicirculi A B C, C G L, Dico lunulas A B C T, C G L F triãgulo A C L æquales eſſe: triangulum enim A C L rectangulum eſt, quare A L poteſt A C, C L, ſed A L, A C, C L ſunt diametri ſemicirculorum, & ſemicirculi ſe habent vt à diametris quadrata, ergo ſemicirculus A C F L ſemicirculis A B C, C G L ęqualis erit, auferantur communes portiones A T C H, C F L K, & ſuperſtites lunulę A B C T, C G L F reliquo triangulo A C L ęquales remanebunt, quod erat oſtendendum.



PROPOSITIO II.

SI ſuper chorda quadrantis alicuius circuli fiat ſemicirculus, lunula à duabus periferijs intercepta, æquabitur triangulo ſub eiuſdem quadrantis chorda & lateribus comprehenſo.

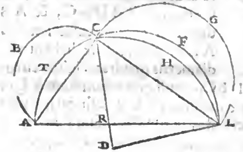
Eſto circuli quadrans A D B C, ſuper cuius chorda A B conſtitutus ſit ſemicirculus A E B; Dico lunulam A E B D triangulo A B C æqualem eſſe; quadratum enim ex A C dupla quadruplum eſt quadrati ex A C, ſed eiuſdem quadrati ex A C duplum eſt quadratum ex A B, quare quadratum ex A C dupla quadrati ex A B quoque duplum erit, eſt tamen A C dupla diameter circuli, cuius quadrans A D B C, & A B diameter circuli, cuius ſemiſsis A E B, & circuli ſe habent vt à diametris quadrata, igitur circulus, cuius quadrans A D B C, circuli, cuius dimidium A E B, duplus erit, cum itaque quadrans A D B C ſit quarta pars circuli dupli, & ſemicirculus A E B dimidium ſubdupli, quadrans A D B C ſemicirculo A E B æqualis erit, dematur communis portio A D B F, & lunula A E B D triangulo A B C æqualis remanebit, quod erat oſtendendum.



PROPOSITIO III.

SI latus maius trianguli ſemicirculo inſcripti, fiat chorda quadrantis alicuius circuli, latus quadrantis triangulum inſcriptum in duo triangula collateralibus lunulis æqualia ſecabit.

Aſſumatur ſchema Prop. primæ, & latus C L fiat chorda quadrantis C D L H, cuius latus C D ſecet A L in R, Dico triangulum C R L lunulæ C G L F; & triangulum A C R lunulæ A B C T adæquari; conſtat ex prima Prop. triangulum A C L valere duas lunulas A B C T, C G L F, itemque ex ſecunda triangulum C D L lunulam C G L H, ergo cum lunula C G L F in vtroque triangulo comprehendatur, & vtrique triangulo commune eſt triangulum C R L, erit triangulum C R L ęquale lunulę C G L F, erat & triangulum A C L ęquale duabus lunulis A B C T, C G L F, ablatis igitur æqualibus lunula videlicet C G L F & triangulo C R L, & triangulum A C R lunulæ A B C T æquale remanebit; Eodem modo probari poteſt æqualitas inter triangulum R D L & lunulam C F L H; cum itaque lunula C G L F triangulo C R L, & lunula A B C T triangulo A C R lateraliter adplicetur, verum eſt triãgula ſuis lunulis collateralibus adæquari, quod erat oſtendendum.



CON-

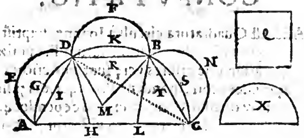
CONSECTARIVM.

Hinc apparet latus quadrantis C D angulum A C L bifariam ſecare, nam cum angulus quadrantis C D L rectus ſit, & latera C D, L D æqualia, angulus D C L ſemirectus erit, ſed rectus quoque eſt angulus A C L, quia in ſemicirculo, ergo reliquus A C D quoque ſemirectus erit, angulus igitur A C L per rectam C D bifariam diuiditur, quod eſt propoſitum.

PROPOSITIO IV.

Datum circulum quadrare.

Proponatur ſemicirculus A G S C, in quo ſemihexagonũ inſcribatur A D B C, & ſuper lateribus fiant ſemicirculi A E D, D F B, B N C, itemque ducta D C angulus A D C à recta D H bifariam ſecetur, eodemque modo fiat ex altera parte, hoc eſt angulus A B C bifariam diuidatur per rectam



B L, & ſuper D B fiat triangulum D B M æquale triangulo D A H vel B L C; Dico rectilineum D H L B M æquari ſemicirculo X, ſuper eiuſdem hexagoni latere deſcripto; Diameter A C poteſt quadruplũ diametri A D, ſed circuli sũt vt à diametris quadrata, ergo ſemicirculus A G S C æquabitur

bitur quatuor ſemicirculis A E D,D F B,B N C & X,auferantur communes portiones A G D I,D K B R,B S C T,& remanebit ſemihexagonum A D B C æquale tribus lunulis A E D G,D F B K,B N C S & integro ſemicirculo X,ſed ex tertia Prop. triangulum D A H æquatur lunulæ A E D G, itemque triangulum B L C lunulæ B N C S, triangulumque D B M ſuæ lunulæ D F B K, cum æquale factum fuerit triangulo D A H vel B L C,& lunulę omnes inter ſe ęquales ſint,igitur ęqualibus magnitudinibus ablatis, hoc eſt à ſemihexagono ablatis tribus triangulis,& ab adgregato lunularum & ſemicirculi demptis tribus lunulis, ſuperſtes rectilineum D H L B M integro ſemicirculo X æquale remanebit, Quare ſi duplo rectilineo D H L B M fiat ęquale quadratum Q, proſiliet quadratum ęquale minori circulo X,ſed ſi deſideretur quadratum ęquale maiori circulo A G S C,quadrupletur quadratum Q. & habebitur quadratum ęquale circulo propoſito A G S C,quadratus igitur eſt circulus,quod erat faciendum.

CONFVTATIO.

HAEc eſt Quadratura circuli à Ioanne Baptiſta Porta excogitata,à nobis quatuor tãtum Propoſitionibus concluſa,quarum duę priores verę ſunt,poſteriores reliquę falſę,& hę quidem correſpondent Prop.4.1.6.17.lib.3.Curuil.element.& quamuis Prop.3.ex qua dependet 4.nempe circuli quadratura,alio forte modo videatur propoſita, quam ipſe facit Prop.6.tamen ſi res diligentius conſideretur, vtraque eundem finem conſequitur, nam intentio autoris nil aliud eſt,niſi demonſtrare triangulum A C R lunulę A B C T,& triangulum C R L lunulę C G L F adęquari, at nobis ſic ipſam proponere viſum fuit, vt Mathematicę methodo magis eſſet conſentanea; Sed videamus quęſo

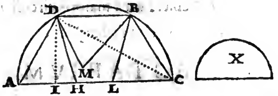
num

num iſthęc quadratura extra limites Archimedęos diuagetur, eſt enim doctrina Archimedis in hac pragmatia, tanquam lapis Heraclius omnium quadratariarum machinationum, quam ſanè diſquiſitionem duplici Propoſitione abſoluemus.

PROPOSITIO I.

DAti circuli quantitatem ſiue aream numeris explicare.

Aſſumatur figura Prop. 4. & à puncto D in diametrum A C deſcendat perpendicularis D I, atque ipſa diameter ſtatuatur 7. erit idcirco latus Hexagoni A D $\frac{7}{2}$ ſed trian-



gulum A D C rectangulum eſt, quare reliquum latus D C erit ℞ $\frac{147}{4}$, cum itaque perpendicularis D I deſcendat ab angulo recto A D C in baſim A C, ſimilia erunt triangula A D I, A D C, quare A D media proportionalis exiſtet inter A C & A I, & obid nota A I $\frac{7}{4}$ itemque D I, ℞ $\frac{147}{16}$, ſed angulus A D C bifariam diuiditur à recta D H, ergo vt C D ad D A ſic C H ad H A, & cōponēdo vt C D plus D A ad D A ſic C A ad H A, erit itaq; H A, ℞ $\frac{147}{4} - \frac{7}{2}$, triangulum igitur D A H notam habebit baſim A H & perpendicu-

larem

cularem D I, non latebit ergo eius area $\frac{147}{16}$ -- ℞ $\frac{7203}{256}$, quę triplata pateſaciet aream trium triangulorum D A H, B L C, D B M, $\frac{441}{16}$ - ℞ $\frac{64827}{256}$; Ad habendam vero areã ſemihexagoni A D B C, ita procedemus, conſtat enim hexagonum omne ordinatum ſub ſemiperimetro & perpendiculari à centro figuræ ad vnum latus ducta; ſed ei perpendiculari æquatur D I, ergo ſi D I ducatur in ſemiperimetrum A D B C, prodibit area totius Hexagoni, ℞ $\frac{64827}{64}$, à cuius ſemiſſe, ℞ $\frac{64827}{256}$, ſi dematur area trium triangulorũ, $\frac{441}{16}$ - ℞ $\frac{64827}{256}$, remanebit rectilineum D H L B M, ℞ $\frac{259308}{256}$ - $\frac{441}{16}$, æquale ſemicirculo X, at ſemicirculus X eſt pars octaua maioris circuli A D B C, quare ſi octies aſſumatur, ℞ $\frac{259308}{256}$ - $\frac{441}{16}$, habebitur area maioris circuli A D B C, ℞ 64827 - 220 $\frac{1}{2}$, quod erat faciendum.

CONSECTARIVM I.

Facile hinc deducitur ratio circumferentiæ ad diametrum, nam cum circulus fuit adinuentus, ℞ 64827 -- 220 $\frac{1}{2}$ diametro exiſtente 7. ſequitur neceſſario latus quadrati circulo ęqualis eſſe Rad. Binomiæ ℞ 64827 - 220 $\frac{1}{2}$ at latus quadrati circulo ęqualis mediat inter ſemidiametrum & ſemiperiferiam, ergo ſi quadratum circulo æquale, ℞ 64827 - 220 $\frac{1}{2}$, diuidatur per ſemidiametrum, $\frac{7}{2}$, orietur ſemicircumferentia, ℞ 5292 - 63, quæ duplata dabit integram circumferentiam, ℞ 21168 - 126. ratio igitur circum-

cumferentiæ ad diametrum eſt, vt ℞ 21168 − 126. ad 7. hoc eſt 20. ad 7. maior quam vera, ſeu 19. ad 7. minor quam vera.

CONSECTARIVM II.

Liquido quoque apparet ratio quadrati à diametro ad ipſum circulum, nam diametro exiſtente 7. erit eius quadratũ 49. at circulus fuit adinuentus ℞ 64827 − 220$\frac{1}{2}$, quare quadratum à diametro ad circulũ ſe habet vt 49 ad ℞ 64827 − 220$\frac{1}{2}$, hoc eſt 49 ad 33$\frac{1}{2}$, maior quam vera, ſeu 49 ad 34$\frac{1}{2}$ minor quam vera.

SCHOLIVM.

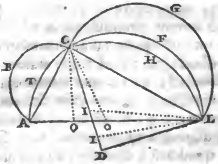
Aſſumpſimus diametrum 7. vt periculum faceremus, num hæc quadratura numeris Archimedęis conſentiret, quod ſanè non contingit, nam demonſtrauit Archimedes circumferentiam ad diametrum maiorem habere rationem, quam 223. ad 71 minorem verò, quam 22 ad 7. ſed ſecundum hanc quadraturam circumferentia ad diametrum, maiorem habet rationẽ, quam 19. ad 7. minorem verò, quam 20. ad 7. ecce quod ratio 20. ad 7. quę maior eſt quam vera, deficit à ratione 223. ad 71. quæ ſecundum Archimedem minor eſt quam vera, prioris enim denominator eſt 2$\frac{6}{7}$ poſterioris verò 3 $\frac{1}{7}\frac{0}{1}$. Eodẽ modo dicendũ eſt de ratione quadrati à diametro ad circulum, nam demonſtrauit Archimedes, quadratũ à diametro ad circulũ, maiorem habere rationem, quam 14. ad 11. minorem verò, quam 284. ad 223. ſed ſecundum hanc quadraturam, quadratum à diametro ad circulum, maiorem habet ratio-

nem, quam 49. ad 34 $\frac{1}{2}$, minorem verò, quam 49 ad 33 $\frac{1}{2}$ ecce quod ratio 49 ad 34 $\frac{1}{2}$, quæ minor eſt quam vera, excedit rationem 284. ad 223, quæ ſecundum Archimedē maior eſt quam vera, prioris enim denominator eſt 1 $\frac{2}{6}\frac{9}{9}$ poſterioris verò 1 $\frac{}{2}$–$\frac{6}{2}$–$\frac{8}{3}$. Quadratura igitur hæc extra limites Archimedis reperitur.

PROPOSITIO II.

IN quibus hæc deficit Quadratura luculenter indicare.

Paulo ante diximus Prop. 3. & 4. falſas eſſe, at 4. falſa eſt ex 3. igitur tertiæ incumbendum, & profectò in eo latet defectus, dum aſſeritur lunulam C G L F triangulo C R L ęqualem eſſe, nam quamuis lunula CGLF in vtroque triangulo A C L, C D L comprehendatur, & vtrique quoque triangulo commune ſit triangulum C R L, non hinc ſequitur triangulum C R L lunulæ C G L F adæquari, nam ſi quis dixerit lunulam C G L F reſpectu trianguli A C L, occupare tantum ſpacium C O L, ſed reſpectu alterius trianguli C D L, ſpacium C I L, corrueret tota Portæ demonſtratio, legitime ergo demonſtratum nō

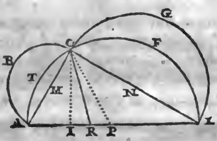


eſt lunulam C G L F triangulo C R L æqualem eſſe, legitime tantum demonſtratum eſt per 1. Prop. triangulum A C L valere duas lunulas A B C T, C G L F, & per 2.

trian-

triangulum C D L valere lunulam C G L H, & nihil amplius.

Sed quæſo manifeſtius per numeros explicemus hæc omnia, hoc eſt indagemus aream triangulorum A C R, RCL & lunarum A B C T, C G L F, nam ſic videbimus num triangulum ACR lunulæ ABCT, & num triangulum R C L lunulæ C G L F æquale ſit; & primo inueſtigemus aream triangulorum, conſtat equidem ex calculo Prop. antecedentis perpendicularem C I eſſe ℞ $\frac{1}{} - \frac{1}{1} - \frac{7}{}$ & rectã A R ℞ $\frac{1}{} - \frac{1}{4} - 7 -$ $\frac{7}{8}$, quare reliqua R L erit $\frac{2\ 1}{2}$ − ℞ $\frac{1\ 4\ 7}{4}$, cũ itaq; triangula A C R, RCL notas habeant baſes A R, R L cum perpendiculari C I, quæ vtrique triangulo communis eſt, non latebunt eorum areæ, eſt ergo



Area trianguli A C R $\frac{147}{16}$ − ℞ $\frac{7203}{256}$

Area trianguli R C L ℞ $\frac{64827}{256}$ − $\frac{147}{16}$.

Pro area verò lunularum veſtiganda ſic procedemus, ex centro P ducta P C, conſtituentur duo triangula A C P, P C L, & duo ſectores PATC, PLFC, triangula inter ſe æqualia ſunt, at ſector P A T C dimidium eſt ſectoris PLFC eſt

Area trianguli A C P, ſiue PCL ℞ $\frac{7203}{256}$

Area ſectoris P A T C $\frac{℞\ 28812 - 147}{4}$

Area ſectoris P L F C $\frac{℞\ 115248 - 294}{4}$

H 2 à qui-

à quibus ſectoribus ſi auferantur triangula A C P, PCL, remanebit.

Segmentum ATCM $\frac{℞\,28812 - 147}{4} - ℞\,\frac{7203}{256}$

Segmentum LFCN $\frac{℞\,115248 - 294}{4} - ℞\,\frac{7203}{256}$

Hoc eſt A T C M $\frac{℞\,5647112}{64} - \frac{147}{4}$

LFCN $\frac{℞\,25930800}{64} - \frac{294}{4}$

Demum inueſtigemus aream duorum ſemicirculorum ABC, LGC hoc modo, eorum diametri notæ ſunt, AC $\frac{7}{2}$ CL $℞\,\frac{1\,4\,7}{4}$ ſed vt diameter AL 7. ad ſuam ſemiperiferiã A C L $℞\,5292 - 63$. ſic erit diameter A C $\frac{7}{2}$ ad ſuam ſemiperiferiam A B C $\frac{℞\,5292 - 63}{2}$. & ſic diameter C L $℞\,\frac{1\,4\,7}{4}$ ad ſuã ſemiperiferiã LGC $\frac{126 - ℞\,11907}{2}$ erit igitur

Area ſemicirculi A B C $\frac{℞\,259308 - 441}{16}$

Area ſemicirculi LGC $\frac{℞\,2333772 - ℞\,1750329}{16}$

Quare ſi à ſemicirculis ABC,LGC auferantur ſegmenta ATCM, LFCN, remanebit.

Lunula ABCT. $\frac{℞\,259308 - 441}{16} - \frac{℞\,5647112}{64} + \frac{147}{4}$

Lunula C G L F. $\frac{℞\,2333772 - ℞\,1750329}{16} - \frac{℞\,25930800}{64} + \frac{294}{4}$

Hoc eſt A B C T. $\frac{℞\,259308}{16} - \frac{℞\,5647112}{64} + \frac{147}{16}$

CGLF. $\frac{℞\,2333772 - ℞\,1750329}{16} - \frac{℞\,25930800}{64} + \frac{147}{2}$

Deuen-

Deuentum eſt ad quantitates adeo irrationales, quod non apparet euidenter quid ſtatuendum ſit de conuenientia triangulorum A C R, RCL & lunularum ABCT, CGLG, expurgemus quæſo eas à vitio fractionis, multiplicentur ergo omnia per 64. & erit hoc modo multiplex.

Triangulum ACR. 588. – ℞ 115248
Triangulum RCL. ℞. 1037232 – 588
Lunula ABCT. ℞ 4148928 – 5647112 + 588
Lunula CGLF. ℞ 37340352 – ℞ 25930800 – 588

Deprimantur ad minores terminos, eas diuidendo per 2. & prodibit,

Triangulum ACR. 294 – ℞ 28812
Triangulum RCL. ℞ 259308 – 294
Lunula ABCT. ℞ 1037232 – ℞ 1411778 + 294
Lunula CGLF. ℞ 9335088 – ℞ 6482700 – 294

Reſoluantur potaſtates, tribuendo tot cyphras numeris abſolutis, quot cyphras conditionarias poteſtatibus ipſis, tribuamus quæſo numeris abſolutis quinque cyphras, & erit poteſtas reſoluta.

Trianguli ACR. $\frac{12425903}{100000}$

Trianguli RCL. $\frac{21522293}{100000}$

Lunulæ ABCT. $\frac{12426323}{100000}$

Lunulæ CGLF. $\frac{21522294}{100000}$

Apparer nunc euidenter maiores eſſe lunulas A B C T, CGLF, quam triangula ACR, RCL, lunula enim ABCT excedit triangulũ ACR in $\frac{420}{100000}$ & lunula CGLF excedit triangulum R C L in $\frac{1}{100000}$ quare ſi æque multiplicia inæqualia ſunt, & partes æqueſubmultiplices inæquales quoque erunt.

Sed

Sed lubet eundem calculum exhibere in via Archimedis, hoc eſt ſupponendo circumferentiam ad diametrum ſe habere, vt 22. ad 7. (quamuis hæc proportiò maior ſit quã vera) indagare conuenientiam præfatorum triangulorum & lunularum, erit ergo ſector PATC $\frac{77}{12}$ ſector PLFC. $\frac{154}{12}$ à quibus ſectoribus ſi auferantur triangula A C P, P C L, remanebit.

Segmentum ATCM $\frac{77}{12} -$ ℞ $\frac{7203}{256}$

Segmentum LFCN $\frac{154}{12} -$ ℞ $\frac{7203}{256}$

Semicirculus A B C erit $\frac{77}{16}$ ſemicirculus L G C. ℞ $\frac{17787}{196}$ Quare ſi à præfatis ſemicirculis ABC, LGC auferantur ſegmenta ATCM, LFCN, remanebit.

Lunula ABCT $\frac{77}{16} - \frac{77}{12} +$ ℞ $\frac{7203}{256}$

Lunula CGLF. ℞ $\frac{17787}{196} - \frac{154}{12} +$ ℞ $\frac{7203}{256}$

Hoc eſt ABCT. ℞ $\frac{7203}{256} - \frac{77}{48}$

CGLF ℞ $\frac{56307}{256} - \frac{77}{6}$

Deuentum quoque eſt ad quantitates & ſi irrationales, non ita tamen laborioſas vt priores, tamen vt euidenter conſtare poſſit de conuenientia triangulorum ACR, RCL, & lunularum ABCT, CGLF, expurgemus quæſo eas à vitio fractionis; multiplicentur ergo omnia per 48. & erit hoc modo multiplex.

Triangulum	ACR.	441 − ℞ 64827
Triangulum	RCL.	℞ 583443 − 441
Lunula	ABCT.	℞ 64827 − 77
Lunula	CGLF.	℞ 506763 − 616

Re-

Reſoluantur poteſtates vt prius, & erit poteſtas reſoluta.

Trianguli	ACR.	$\frac{18638854}{100000}$
Trianguli	RCL.	$\frac{32283440}{100000}$
Lunulæ	ABCT.	$\frac{17761146}{100900}$
Lunulæ	CGLF.	$\frac{9587288}{100000}$

Apparet nunc euidenter ex hoc calculo minores eſſe Lunulas A B C T, C G L F, quam triangula A C R, RCL, triangulum enim A C R excedit lunulam A B C T in $\frac{8}{1}\frac{7}{0}\frac{7}{0}\frac{7}{0}\frac{0}{0}\frac{8}{0}$, & triangulũ RCL excedit lunulam CGLF in $\frac{2}{}\frac{2}{1}\frac{6}{0}\frac{9}{0}\frac{6}{0}\frac{1}{0}\frac{5}{0}\frac{2}{}$, & ſic nunc totum oppoſitum adinuenimus quam prius, in dubium itaque per calculum res redacta eſt ob irrationales quantitates, quarum valor in numeris abſolutis exacte dari non poteſt, tamen ſatis conſtare arbitror nullam æqualitatem reperiri inter præfatas lunulas & triangula, qualiſcunque aſſumatur calculus, quod erat à nobis inquirendum.

Attamen vt ſtudioſis morem geramus, non grauabimur per aliam viam experiri num triangula lunulis adæquentur, ſupponamus quæſo triangula lunulis adæquari, & accipiamus triangulum ACR & lunulam ABCT ex primo calculo, quare ſi triangulum ACR æquatur lunulæ ABCT

$\frac{147}{16}$ − ℞ $\frac{7203}{256}$ æquabitur $\frac{147}{16}$ + ℞ $\frac{259308}{256}$ − ℞ $\frac{5647112}{4095}$

& per antitheſim

$\frac{147}{16}$ − $\frac{147}{16}$ + ℞ $\frac{5647112}{4096}$ æquabitur ℞ $\frac{259308}{256}$ + ℞ $\frac{7203}{256}$

hoc eſt ℞ $\frac{5647112}{4096}$ æquabitur ℞ $\frac{17294403}{256}$

quadrentur omnia, &

hoc

$$\frac{5647112}{4096} \text{ æquabitur } \frac{17294403}{256}$$

hoc eſt 1378 $\frac{8}{9}\frac{5}{1}\frac{1}{2}$ æquabitur 67556 $\frac{1}{5}\frac{6}{5}\frac{7}{6}$ quod eſt falſum, maior eſt enim quantitas 67556 $\frac{1}{5}\frac{6}{5}\frac{7}{6}$ quam 1378 $\frac{8}{9}\frac{5}{1}\frac{1}{2}$. Accipiamus quoq; lunulam A B C T ex ſecundo calculo, quare ſi triangulum A C R æquatur lunulæ ABCT,

$$\frac{147}{16} - ℞ \frac{7203}{256} \text{ æquabitur } ℞ \frac{7203}{256} - \frac{77}{48}$$

& per antitheſim

$$\frac{147}{16} + \frac{77}{48} \text{ æquabitur } ℞ \frac{7203}{256} + ℞ \frac{7203}{256}$$

hoc eſt $\frac{8288}{768}$ æquabitur ℞ $\frac{28812}{256}$

quadrentur omnia, &

$$\frac{68690944}{589824} \text{ æquabitur } \frac{28812}{256}$$

hoc eſt 116 $\frac{6}{1}\frac{0}{1}\frac{5}{5}\frac{5}{2}$ æquabitur 112 $\frac{3}{6}\frac{5}{4}$ quod eſt falſum, maior eſt enim quaantitas 116 $\frac{6}{1}\frac{0}{1}\frac{5}{5}\frac{5}{2}$ quam 112 $\frac{3}{6}\frac{5}{4}$. Ex vtroq; itaq; calculo apparet euidenter lunulam ABCT triangulo ACR non adæquari, quod etiã dicendum eſſet de lunula CGLF & triangulo RCL, ſi ad eandẽ praxim reuocarentur, ſed ſufficit de triangulo ACR & lunula ABCT oſtenſionem feciſſe, triangula ergo ACR RCL non adæquantur lunulis ABCT, CGLF.

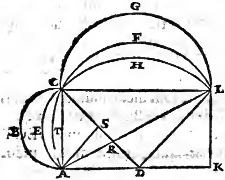
Sed ſi fortaſſis aliqui conquerantur, me auctoris demõſtrationem euertiſſe, ſatisfaciamus eis, & propriam, quam ipſe affert Prop. 6. integre proferamus.

PROP. VI. Curuil. Element.

Duas lunulas inæquales in ſemicirculi ambitu deſcriptas ſeorſum quadrare.

Eſto

1 **E**Sto rectangulum triangulum A C L. cuius porrectius latus CL ſit duplum exilioris AC, & circumferantur lunulæ ex more, quibus adijce ſuas literas CGLF & ABCT, mox parallelogrammum conſtituatur ex lateribus AC, CL, & ſit AKCL, & fiat quadrans circuli CDLH, & ſubdupli A E C S, nos rationem reddituri, lineam CR triangulum A C L partiri taliter, vt trianguli compares mutuo correſpondentes & æquales ſint, vt A C R par ſit RDL, & triangulum ACR par ſit lunulæ ABCT, & triangulum CRL ipſi CGLF, Quoniã lunulæ CGLF, & ABCT



pares ſunt triãgulo ACL quarta huius noſtri adiuuante, & triãgulo ACL par trimetrum CDL, quoniam vtrumq; ſui parallelogrammi dimidium eſt, vt figura 4. huius demonſtratũ eſt, ergo triãgulum C D L eſt duabus præſignatis iam lunulis CGLF, & ABCT æquale, ſed triãgulum CDL eſt æquale lunulæ CGLH, ergo lunula CGLH eſt æqualis CGLF, & ABCT, ſubducatur ſemilunula CGLF, vtpote vtriq; communis, remanet ſublunula CFLH æqualis A B C T, & quemadmodum triangulum CDL æqualæ ACL, ſubducatur commune CRL, reliquum triangulum RDL reliquo triangulo ACR æquale, lunularum partium repræſentantia, ſequitur triangulum RDL eſſe æquale ſublunulæ CFLH, & triangulum A C R æquale lunulæ ABCT, à quo ſi triangulum A C S ſubtrahatur, æquale lunulæ A B C E per primã huius, remanet ſubtriangulum ASR imæ lunulæ A E C T par, quod erat demonſtrandum.

2 Vel hoc modo, ſi triangulum ACL ęquale eſt lunulis ABCT, CGLF, compleatur triangulum CDL, & compleatur perfecta lunula, quæ ſit CGLH, ergo addita pars trianguli RDL ęqualis erit additæ lunulæ CFLH, ſed pars trianguli addita RDL ęqualis eſt triangulo ACR, vt vidimus, & par eſt lunula ABCT lunulę CFLH.

3 Vel hoc modo, tres lunulę ABCE, CGLF, AECT ſunt æquales trigono ACL, & duæ lunulę CGLF, CFLH trigono CDL, ergo omnes quatuor iam dictę lunulæ ſunt æquales duobus trigonis A C R & C D L, ſed tres lunulæ ABCE, CGLF, LFCH ſunt æquales trigono ALK, ergo reliqua lunula AECT eſt ęqualis trigono ASR, quod quęrebamus.

4 Vel tres lunulæ ABCE, AECT, CGLF æquales ſunt trigono ACL, & trigonum C D L æquale lunulæ CGLH, tolle triangulum C D L æquale iam dictæ lunulæ CGLH, reliquum triangulum ACR lunulis ABCE, AECT æquale eſt, tolle lunulam perfectam ABCE, ſublunula AECT ſubtriangulo ASR æqualis erit, quod erat demonſtrandū.

1 IN hac Prop. 6. quatuor adducuntur probationes, ex prima bene deducitur lunulas C F L H, A B C T æquales eſſe, itemq; triangula RDL, ACR inuicem æqualia, ſed quomodo triangulum R D L ſublunulæ CFLH, & triangulum ACR lunulæ ABCT æquale ſit, nō video, neq; verum eſt ſubtriangulo ASR imam lunulam AECT parem eſſe, nam hoc tunc ſequeretur, ſi lunulæ ABCT par eſſet triangulum A C R, quod quia indemonſtratum eſt, indemonſtratam quoque cernimus æqualitatem inter ſubtriāgulum A S R, & imam lunulam AECT, & aduerſus hanc probationem locum habet inſtātia ſuperius à nobis facta, nam ſupponit autor ſe rite demonſtraſſe triangulum C R L par eſſe lunulæ CGLF, quod nequaquam præſtitit.

2 Ex

2 Ex ſecunda verum eſt triangula RDL, ACR eſſe æqualia, itemque lunulas ABCT, CFLH, ſed non ſecundum illud additamentum, quod ex noſtra inſtantia mox euaneſcit, ſed bene verum ex prima, in qua cum idem probauerat & bene, à ſecunda probatione & mala ſuperſedere poterat.

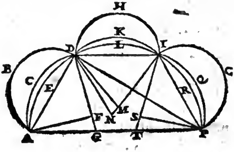
3 Quod affert in tertia non bene ſe habet, nam lunula CGLF bis aſſumenda eſt, & ita formari argumentum, tres lunulæ ABCE, AECT, CGLF æquantur trigono A C L, itẽq; duæ CGLF, CFLH trigono CDL, ergo quinque lunulæ ABCE, AECT, CGLF bis, CFLH æquales ſunt duobus trigonis ACL & CDL, ſed tres lunulæ ABCE, CGLF; CFLH æquantur duobus trigonis ACS, CDL, ergo reliquæ duæ AECT, CGLF duobus reliquis triangulis ASR, CRL æquales remanebunt, ſed non conſtat triangulum ASR æquari lunulæ AECT, vel triangulum CRL lunulæ CGLF, ergo. At autor in ſuo argumento quatuor tantũ facit lunulas, nam ſupponit lunulam CGLF parem eſſe triangulo C R L, vt ſæpius diximus, quod nunquam probauit.

4 In idem abſurdum incidit quarta probatio, nam tres lunulæ ABCE, AECT, CGLF pares ſunt trigono ACL, & lunula CGLH trigono CDL, ablatis trigono CDL & ſua compari lunula CGLH, remanent rurſus tres lunulæ priores ſuo compari triangulo ACL æquales, quod argumentum videtur principium petere, cum id probet, quod prius conſtabat; ſed ipſe aliter proponit, putans triangulo CRL parem eſſe lunulam CGFL vt moris, & ob id concludit lunulas ABCE. AECT ęquari triangulo ACR, & hinc ablatis æqualibus nempe lunula ABCE & triangulo ACS, pronunciauit ſublunulam AECT ſubtriãgulo ASR parẽ eſſe.

Hæ ſunt quatuor probationes autoris, quæ vt vidimus non bene ſe habent, & profectò in eo lapſus eſt Porta,

quod firmiter putauerit lunulam CGLF triangulo CRL omnino parem eſſe, quod ſanè nunquam probauit, ex quo vnico & generali lapſu, reliqui defectus in determinanda ſubtriangulorum & ſublunularum paritate inſurrexerunt.

Sed prætereundus non eſt latentior quidam alter defectus; nam Prop.6. triangulum fabricat rectangulum, cuius duo latera circa rectum ſunt in ratione dupla, CL enim duplum eſt ad AC, poſtmodum Prop.17. agens de circulo quadrando, mutat trianguli ſpeciem, vt appparet ex figura præfatæ Propoſit. 17. nam triangulum rectangulum ADP



non habet latera circa rectum in dupla ratione, eſt enim DP minus quam duplum ipſius AD, & tamen ex Prop.6. dependet Propoſ.17. nempe ipſum quadrandi circuli artificiũ, & hinc omnia ſuperius comprobata ſymptomata corruunt.

Quod latus D P minus ſit quam duplum ipſius A D, apparet manifeſto, nam A D latus hexagoni æquatur ſemidiametro, cuius duplum eſt diameter A P, ſed D P diametro minus eſt, ergo DP minus erit quam duplum ipſius AD, patet & ſecnndo, nam DP eſt latus trianguli æquilateri, cum ſubtendat trientem periferię, at latus trianguli ęquilateri potentia triplum eſt ad ſemidiametrum, igitur latus D P lõgitudine minus erit quam duplum ipſius AD.

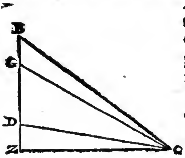
SCHOLIVM.

VT autori atque alijs ſatisfaceremus adduximus integram eius Prop. 6. & iam liquido apparuit, inſtantiam

tiam, qua noſtra Prop. 3. defecit, etiam aduerſus ſuam 6. locum habuiſſe, vt ſuperuacaneum fuit ipſam adduxiſſe, imo non ſolum Prop. noſtra eadem complectitur, quæ in ſua ſunt, ſed vt Mathematico ſtylo conformior fieret catholice propoſita eſt, ipſe enim ſingulare & conditionatum propoſuit triangulum rectangulum, cuius duo latera circa rectum ſunt in ratione dupla, & ob id lunulæ CFLH. ABCT inuicem adæquantur, necnon & triangula ipſa ACR, RDL, ſed laterum circa rectum ratione commutata, æqualitas & inter lunulas & inter triangula prorſus euaneſcit.

EXERCITATIO QVINTA.

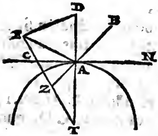
NON bene ſe geſſit Egnatius Dātes ad theorema ſeptimum optices Euclidis, & ad vigeſimum de ſpeculis, in ſeptimo theoremate dicit Euclides, quod ſi duæ magnitudines æquales BG, DZ conſpiciantur ab oculo in C, maior apparebit magnitudo DZ quam BG, quia angugulus DCZ maior eſt angulo BCG.



Ad quod oſtendendum ſic ratiocinatur Egnatius, angulus CDG maior eſt angulo CZD per.32.primi, at angulus CGD maior eſt angulo CDZ per.19. primi, quia latus C D, quod opponitur angulo CGD, maius eſt latere CZ, quod opponitur an-

angulo CDZ, ergo angulus DCZ angulo GCD maior erit, eodem modo probabitur, quod angulus B C G minor ſit angulo GCD, ergo multomagis B C G minor erit angulo DCZ: Miror ſanè quomodo hic vir alioqui diligentiſſimus in re tam leui ceſpitauerit, Prop. enim 19. ſeu potius 18. primi locum habet in vnico triangulo, quando eius anguli inuicem comparantur, quod ille videlicet angulus in aliquo triangulo maior dicendus eſt, qui à maiore latere ſubtenditur, at ipſe aſſumit duo triangula GCD, DCZ, ex qua Propoſitione male intellecta falſum concluſit, quod angulus CGD maior ſit angulo CDZ, imo oppoſitum angulus CDZ maior eſt angulo C G D, externus interno in triangulo G C D per. 16. primi, vel per 32. qua ipſe vtitur: Quod autem angulus D C Z maior ſit angulo B C G legitime demonſtrauit Vitellio lib. 4. ſuæ optices Prop. 24. quem conſulere poterat.

Eodem modo lapſus eſt Egnatius ad theorema 20. de ſpeculis, dum oſtendere nititur rectam E A, quia opponitur angulo obtuſo ECA maiorē eſſe recta AZ, quę opponitur angulo acuto A C Z, in qua argumentatione veniunt duo triangula E A C, C A Z, quæ nullo modo admittuntur à Prop. 18. ſeu potius 19. primi, cum illa locum tantum habeat in vnico triangulo, vt paulo ante monuimus.



In pfato theoremate 20. aſſeritur, quod angulus E C A ſit obtuſus, & quod recta EA maior ſit quam A Z; Quod angulus E C A ſit obtuſus, ex eo fit manifeſtum, quia angulus T A C eſt rectus, fit enim à ſemidiametro T A & tangente N C, ergo angulus T C A erit acutus, & ſuus deinceps E C A obtuſus; Ad oſtendendum vero quod recta E A maior ſit quam

quam AZ, conſideranda ſunt duo triangula E C A, C A Z, cum itaque angulus E C A maior ſit quam A C Z, & C A Z, C A E æquales, erit C Z A maior quã C E A, ergo in triã gulo EZA angulus Z maior erit quam angulus E, & obid latus EA latere AZ maius, quod erat oſtendendum.

EXERCITATIO SEXTA.

CVM incidiſſem in Arithmeticam Maurolyci Venetijs impreſſam anno 1575. prope finẽ libri primi duas Propoſitiones reperi mutilas & imperfectas, nempe 64. & 65. imo 65. caret principio, quæ, vt ibi dicitur, ſic erant in exemplari manuſcripto, placuit præfatas Propoſitiones reſtituere, ne nobilis ille labor Maurolyci ſic imperfectus cerneretur, quod ex veſtigijs ab eo derelictis mihi non admodum arduum fuit.

PROPOSITIONES MVTILAE

Prop. 64.

OMnis columna pentagona cum duplo quadrati collateralis ſimul ſumpta, triplum valet ſuæ pyramidis pentagonæ.

Exempli gratia, columna pentagona quinta 175. cum duplo quadrati quinti 25. hoc eſt 50. facit 225. quod triplum eſt ipſius

Per. 43.	
175.	125. cubus 5.
cum col. ⬠ 5.	40. co. △. 4.
	10. △. 4.

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per 11. 25 □ 5. □ 5 25.	15. △ 5. 10. △. 4.	

ſius pyramidis pentagonæ quintæ 75. quod oſtenditur ſic; columna pentagona quinta æqualis eſt cubo quinto per 43. columnæ triangulæ quartę, & triangulo quarto ſimul acceptis, quibus appono vnum quadratum quintum, & pro altero quadrato quinto appono duos triangulos quintum & quartum, qui per 11. ſimul

*

Hic pauca deſunt

demonſtrandum erit quod * na triangula quarta, triangulo quinto, * duobus triangulis quartis; ſimul triplum eſt pyramidis pētagonæ quintę, ſed pyramis pentagona quinta per 36. conſtat ex combinatione duarum pyramidū, ſcilicet quadratę quintę, & triangulę quartę; ergo eſt demonſtrandum, quod dictum aggregatum eſt triplum huic combinationi, quod ſic patet, vna pars illius aggregati, ſcilicet cubus quintus cū quadrato quinto & triāgulo quinto ſimul per prędecentem, ęqualis eſt triplo quintę quadratę pyramidis

Pyr. ⬠. 5. 75.	Pyr. □ 5. 55. Pyr. △ 4. 20.

*

Hic multa deſunt, quę non ſunt in exemplari manuſcripto

per. 37. * ſcilicet pentagonę quintæ

Per. 37.
95
Pyr.hex. 5.

Pyr. ⬠. 5.
75.
Pyr. △. 4.
20.

tæ * Quare oſtendẽdum eſt, quod ſupradictum aggregatum eſt triplum huius combinationis, quod conſtabit ſic. Vna pars illius aggregati, ſcilicet columna pentagona quinta cum duobus quadratis quintis per præcedẽtem æquiualet triplum pyramidis pentagonæ quintæ, quę fuit vna pars combinationis, & ſimiliter reliqua pars aggregati, ſcilicet columna triangula quarta cum duobus triangulis quartis ſimul per. 50. huius, triplum valet pyramidis triangulæ quartæ, quod eſt reſiduum combinationis. Quamobrem quoniam duæ partes aggregati duabus partibus combinationis, ſingulæ ſingulis triplæ ſunt, propterea per primã quinti elementorum, & totum aggregatum totius combinationis triplum valebit, quod fuit demonſtrandum, & eodem ſyllogiſmo pro quouis alio aſſignato loco vtemur ad roborationẽ propoſiti.

PROPOSITIONES RESTITVTAE

Prop. 64.

OMnis columna pentagona cum duplo quadrati collateralis ſimul ſumpta, triplum valet ſuæ pyramidis pentagonæ.

Exempli gratia, columna pentagona quinta 175. cum duplo quadrati quinti 25. hoc eſt cum 50. facit 225. quod triplũ eſt ipſius pyramidis pẽtagonæ quintæ 75. quod oſtenditur ſic; columna pentagona quinta æqualis eſt cubo quinto per 43. columnæ triangulæ quartę, & triangulo quarto ſimul acceptis, quibus appono vnum quadratum quintum, & pro altero quadrato quinto appono duos triangulos quintum & quartum, qui per 11. ſimul ſunt æquales quadrato quinto; Nunc demõſtrandũ erit, quod cubus quintus cum quadrato quinto; columna triangula quarta, triangulo quinto, & duobus triangulis quartis, ſimul triplum eſt pyramidis pẽtagonæ quintæ. Sed pyramis pentagona quinta per 36. conſtat ex combinatione duarum pyramidũ, ſcilicet quadratę quintę, & triangulę quartę; ergo eſt demonſtrandum,

Per. 43.

175.	125. cubus 5.
col. ⬠ 5.	40. co. △. 4.
	10. △. 4.

per 11.

25	15. △ 5.
□ 5.	10. △. 4.

□ 5
25.

Per. 36.

75.	Pyr. □ 5.
Pyr. ⬠. 5.	55.
	Pyr. △ 4.
	20.

dum, quod dictum aggregatum eſt triplum huic combinationi, quod ſic patet, vna pars illius aggregati, ſcilicet cubus quintus cũ quadrato quinto & triãgulo quinto ſimul per præcedentem, æqualis eſt triplo quintæ quadratæ pyramidis, ſed per 50. columna triãgula quarta cum duobus triangulis quartis æqualis eſt triplo pyramidis triangulæ quartæ, ergo per primam quinti elementorum congeries cubi, quadrati, & trianguli quinti, vna cum colũna triangula quarta & duobus triangulis quartis, triplum erit ad totam combinationem duarum pyramidum, ſcilicet quadratæ quintæ & triangulæ quartæ, quandoquidem partes ſingulæ partibus ſingulis triplæ ſunt, quod erat demonſtrandum, & ita in ſimilibus faciemus.

Propoſitio 65.

OMnis columna hexagona cum duplo quadrati collateralis ac præcedenti triangulo triplum valet ſuæ pyramidis hexagonæ.

Exempli gratia, colũna hexagona quinta 225. cũ duplo quadrati quinti 25. hoc eſt cum 50. & triangulo quarto 10. facit 285. quod triplum eſt ipſius pyramidis hexagonæ quintę 95. quod oſtenditur ſic, columna hexagona quinta per 44. æqualis eſt columnæ pentagonę quintæ, columnæ triãgulę quartę & triangulo quarto ſimul acceptis, quibus appono duos quadratos quintos & triangulum quartum, oſtendendum eſt igitur quod columna pentagona quinta cum columna triangula quarta, duobus quadratis quintis & duobus triang. quartis ſimul triplũ eſt pyramidis hexagonę quintæ, ſed pyramis hexagona quinta per 37. conſtat ex combinatione duarum pyramidum, ſcilicet pentagonæ quintę & triangulæ quartæ, Quare oſtendendum eſt, quod ſupradictum aggregatum eſt triplum huius combinationis, quod con-

Per. 44.
225.
col. hex. 5.

175. col. ⬠. 5
40. col. △. 4
10. △. 4.
25. □ 5.
25. □ 5.
10. △. 4.

Per. 37.
95.
pyr. hex. 5.

Pyr. ⬠ 5.
75.
Pyr. △ 4.
20.

conſtabit ſic, vna pars illius aggregati, ſcilicet columna pentagona quinta cũ duobus quadratis quintis per præcedentem æquiualet triplũ pyramidis pentagonæ quintæ, quæ fuit vna pars combinationis, & ſimiliter reliqua pars aggregati, ſcilicet columna triangula quarta cum duobus triangulis quartis ſimul per 50. huius triplum valet pyramidis triangulæ quartæ, quod eſt reſiduum combinationis. Quamobrem quoniam duæ partes aggregati duabus partibus combinationis, ſingulæ ſingulis triplæ ſunt, propterea per primam quinti elementorum, & totum aggregatum totius combinationis triplum valebit, quod fuit demonſtrandum, & eodem ſyllogiſmo pro quouis alio aſſignato loco vtemur ad roborationem propoſiti.

Exer-

EXERCITATIO SEPTIMA.

IMpreſſum eſt Pariſijs anno 1615. nobile opus at poſthumum Franciſci Vietæ de Aequationum recognitione & emendatione, in quo perlegendo animaduerti quædã & ſi leuiſſima deſiderari.

Ad cap. 6. de Emendatione Aequationum Probl. 3. deeſt caſus quartus.

Et ſi proponatur

$$\left.\begin{array}{l}\text{A quad. quad.}\\ -\text{ G plano in A quad. bis}\\ -\text{ B ſolido in A}\end{array}\right\}\text{ æquari Z plano - plano}$$

Radix plana effingedi quadrati ſtatuetur

$$\frac{\text{B ſolidum}}{\text{E bis}}+\text{E in A}\ \Big\}\text{ comparanda }\left\{\begin{array}{l}\text{A quadrato}\\ +\text{ E quadrato}\ \frac{1}{2}\\ -\text{ G plano}\end{array}\right.$$

Ad idem Problema 3. aliter demonſtratum deſunt quatuor caſus, ſcilicet 3. 4. 5. 6.

Et ſi proponatur

$$\left.\begin{array}{l}\text{A quad. quad.}\\ -\text{ G plano in A quad.}\\ +\text{ B ſolido in A}\end{array}\right\}\text{ æquari Z plano - plano}$$

Radix plana effingendi quadrati ſtatuetur.

$$\frac{\text{B ſolidum}}{\text{Lv}\left\{\begin{array}{l}\text{[illegible] planum 4.}\\ +\text{G plano4.}\end{array}\right.}-\text{Lv}\left\{\begin{array}{l}\text{E plano}\\ +\text{ G plano}\end{array}\right\}\text{ in A quad.}$$

$$\text{comparanda}\left\{\begin{array}{l}\text{A quadrato}\\ +\text{ E plano}\ \frac{1}{2}\end{array}\right.$$

Et ſi

Et ſi proponatur

$$\left.\begin{array}{l}\text{A quad. quad.}\\ -\text{G plano in A quad.}\\ -\text{B ſolido in A}\end{array}\right\}\text{ æquari Z plano-plano}$$

Radix plana effingendi quadrati ſtatuetur

$$\left.\frac{\text{B ſolidum}}{\text{Lv}\left\{\begin{array}{l}\text{E planum 4.}\\ +\text{G plano 4.}\end{array}\right.}+\text{Lv}\left\{\begin{array}{l}\text{E plano}\\ +\text{G plano}\end{array}\right.\right\}\text{ in A quad.}$$

$$\text{comparanda}\left\{\begin{array}{l}\text{A quadrato}\\ +\text{E plano}\tfrac{1}{2}\end{array}\right.$$

Et ſi proponatur

$$\left.\begin{array}{l}\text{G planum in A quad.}\\ +\text{B ſolido in A}\\ -\text{A quad. quad.}\end{array}\right\}\text{ æquari Z plano-plano}$$

Radix plana effingendi quadrati ſtatuetur

$$\text{Lv}\left\{\begin{array}{l}\text{E planum}\\ +\text{G plano}\end{array}\right\}\text{ in A quad.}+\frac{\text{B ſolidum}}{\text{Lv}\left\{\begin{array}{l}\text{E planum 4.}\\ +\text{G planum 4.}\end{array}\right.}$$

$$\text{comparanda}\left\{\begin{array}{l}\text{E plano}\tfrac{1}{2}\\ +\text{A quadrato.}\end{array}\right.$$

Et ſi proponatur

$$\left.\begin{array}{l}\text{G planum in A quad.}\\ -\text{B ſolido in A}\\ -\text{A quad. quad.}\end{array}\right\}\text{ æquari Z plano-plano}$$

Radix plana effingendi quadrati ſtatuetur

$$\text{Lv}\left\{\begin{array}{l}\text{E planum}\\ \text{G planum}\end{array}\right\}\text{ in A quad.}-\frac{\text{B ſolido}}{\text{Lv}\left\{\begin{array}{l}\text{E planum 4.}\\ \text{G planum 4.}\end{array}\right.}$$

$$\text{comparanda}\left\{\begin{array}{l}\text{E plano}\tfrac{1}{2}\\ +\text{A quad.}\end{array}\right.$$

EXERCITATIO OCTAVA

QVæſtio 19.lib.2. Arithmeticorum Diophāti Alexandrini mutila & imperfecta ad nos peruenit, nam ſecundum eius Zeteſim propoſita quæſtio non ſoluitur, nec dicendum eſt, imo ne ſuſpicandum quidem erraſſe Diophantum, nam ſanè quæſtionem non propoſuiſſet, ſi ſolutionem ignoraſſet; Hanc quæſtionem vnica tantum hypoſtaſi, vt ex eius veſtigijs apparet, abſoluiſſe Diophantum credo, quamuis neminem ante nos; qui vnica hypoſtaſi ſolutionem tradiderit, mihi videre contigit, tentauit id Gulielmus Xilander, ſed eius numeri maxime deprauati ſunt, & quæſtioni non ſatiſfaciunt, ſcio equidem Simonem Steuinum tomo 5. Mathem. Hypomnematum in principio ſoluiſſe quæſtionem per ſecundas radices ſiue ex duplici hypoſtaſi, noſtram ſolutionem ex vnica hypoſtaſi elaboratam iam publicauimus in quodā libello Venetijs impreſſo anno 1613. anno vero 1621. editus eſt Pariſijs, Diophantus Græce & Latine à Gaſpare Bacheto diligenter & interpretatus & abſolutiſſimis commentarijs illuſtratus, in quo Bachetus Xilandri Zeteſim reſtituit, ac ſuam aliam longe faciliorem adiunxit, placuit has omnes ſolutiones ob earum diuerſitatem denuo euulgare, vt huius artis ſtudioſi ſummam capiant delectationem.

QVAE-

Q V AE S T I O.

DAtus numerus in tres diuidatur, quorum quiſque proxime ſequenti vbi dederit ſui partem quæ imperatur, & aliquot item vnitates, datis acceptiſque vt mandatum fuit omnibus, æquales diuiſi partes exiſtant; Hoc pacto diuidendus ſit numerus 80. in tres alios, vt primus ſui quintatem ac 6. ſecundo, ſecundus ſui ſextantem & 7. tertio, tertius ſui ſeptantem & 8. primo det, itaque vltro citroq; datis & acceptis ex præſcripto omnibus, æqualitas exiſtat.

Zeteſis Diophanti corrupta.

STatuemus primum 5 N, ſecundum 12. ergo ſecundus vbi quintantem primi ac 6. acceperit, erit 1 N + 18. verum hic ſecundus puta 12. vbi ſui ſextantem 2. & præterea 7. ideſt in ſumma 9. amiſerit, dum ijs tertiũ impertit, retinet 3. & à primo donatus deinde 1 N + 6. habet ſummam 1 N + 9. tantumdem ergo reliqui etiam dato receptoq; quod imperabatur habebunt. At primo ijs quæ iam diximus expẽſis, reſtabant 4 N – 6. vt ergo habeat 1 N + 9. deſunt ei adhuc 15 – 3 N, hoc ergo eſt ſeptans tertij & præterea 8. quare 8. hinc ſublatis, quod reſtat 7 – 3 N. ſeptans erit tertij, ergo tertius 49 – 21 N, reſtat vt is det & recipiat quæ iubetur, Atqui re peracta habet ſummam 43. – 18 N, quæ æquatur. 1 N + 9. fit 1 N $\frac{1}{1}\frac{4}{9}$, eſt ergo primus $\frac{1}{}\frac{7}{1}\frac{}{9}$ ſecundus. $\frac{2}{}\frac{3}{1}\frac{8}{9}$ tertius $\frac{2}{}\frac{1}{1}\frac{7}{9}$.

S C H O L I V M.

EX præfata Zeteſi, quam etiam ſequitur Scholiaſtes, non ſoluitur queſtio, etenim inter omnes partes vltro

citroque acceptis & datis æqualitas exiſtit, nempe $\frac{156}{19}$ ſed ipſarum ſumma $\frac{615}{19}$ non conſtat 80. ſed $32\frac{7}{19}$

Zeteſis Steuini.

Statuemus primum 1 N, iam vero quia ad poſitionem ſecundi aditus ſit intricatior, reſq; hæc ſit operoſa & ardua, vt ea ſtatuatur in characteribus primo homogeneis, hoc eſt 1 N, atq; id alio compendio indagandum ſit, ponemus pro ipſo 1. ſecund. N. quare conſequens eſt tertium numerum eſſe − 1 N − 1 ſecund. N + 80. atq; ſi tribus his numeris zetematis leges relegas atque æmuleris, videlicet de ſumma ſecundi & $\frac{1}{5}$. primi numeri + 6. ſubducas $\frac{1}{6}$. ſecũdi + 7. ſupererit reliquus primus $\frac{5}{6}$ ſecund. N + $\frac{1}{5}$ N − 1. & rurſum de ſumma tertij & $\frac{1}{6}$ ſecundi + 7. ſubducas $\frac{1}{7}$ tertij + 8. ſupererit reliquus ſecundus − $\frac{6}{7}$ N − $\frac{29}{42}$ ſecund. N + $\frac{473}{7}$. hi duo reliqui inter ſe ex ſententia quæſtionis æquales erunt, qua æqualitate ſecundum artis præcepta reducta tandem inueniemus 1 ſecund. N, æqualem − $\frac{111}{160}$ N + 45. Reuocato igitur valore 1 ſecund. N [quæ optatæ partitionis pars erat altera] ad numeros figuratos poſitionis prioris − $\frac{111}{160}$ N + 45. operis exegeſim denuo iteremus, hoc modo, prima optata numeri 80. pars 1 N ſecunda − $\frac{111}{160}$ N + 45. iam vt tertia pars conſtituatur, inueniendus arte eſt valor − 1 ſecund. N, quæ in numero ordinis tertio vſurpatur, is autem inuenitur ex proportione 1 ſecund. N, dat. − $\frac{111}{160}$ N + 45. ergo − 1 ſecund. N, quid dabit? concludes $\frac{111}{160}$ N − 45 iſte numerus in locum − 1 ſecund. N. ſubrogatus, & cum cæteris − 1 N. & + 80. compoſitus conflabit numeri 80. optatã partem tertiam − $\frac{49}{160}$ N + 35. Atque ſi iſtis tribus numeris propoſiti zetematis leges ſequaris, videlicet ſi de ſumma ſecundi & $\frac{1}{5}$ primi + 6. ſubducas $\frac{1}{6}$ ſecundi + 7.

ſu-

ſupererit reliquus primus — $\frac{2}{1}\frac{2}{2}\frac{1}{0}$ N + $\frac{7\ 1}{2}$ ſimili ratione inuenietur ſecundus reliquus — $\frac{1}{1}\frac{2}{2}\frac{1}{0}$ N + $\frac{7\ 1}{2}$ & tertius reliquus — $\frac{1}{1}\frac{2}{2}\frac{1}{0}$ N + $\frac{7\ 1}{2}$. Quamobrem cum tertius iſte reliquus vtrilibet antecedentium æquetur, videlicet + $\frac{1}{1}\frac{2}{4}\frac{1}{0}$ N + 7. æqualia ſint ipſis — $\frac{1}{1}\frac{2}{0}\frac{1}{0}$ N + $\frac{7\ 1}{2}$ reducta æqualitate inueniemus tandem 1 N valere $2\frac{4}{5}\frac{4}{6}\frac{0}{1}$ pro parte prima, & — $\frac{1}{1}\frac{1}{0}\frac{1}{0}$ N + 45. valere $\frac{2}{1}\frac{7}{6}\frac{8}{1}\frac{6}{}$ pro parte ſecunda, denique — $\frac{4}{1}\frac{}{6}\frac{9}{0}$ N + 35. valere $\frac{9}{1}\frac{8}{6}\frac{1}{1}\frac{4}{}$. pro parte tertia, inuenti iſti numeri ſimul compoſiti 80. conflant, & propoſitę quæſtionis leges implent, nam ex zetematis ſententia ſinguli vt decet relinquunt. $\frac{2}{1}\frac{4}{1}\frac{8}{}\frac{0}{1}$ quamobrem 80. diuiſus eſt in partes tres & cæt.

Zeteſis Xilandri à Bacheto reſtituta.

CVM tres numeri qui quæruntur 80. ſummam conficiant, neque dum eorum partes adduntur detrahunturq; huic ſummæ quicquam decedat, cum quod vni aufertur, alteri adijciatur, & nihil excidat amittaturue, intelligere licet, æqualitatem trium numerorum vltimo exiſtentem eam fore, vt quiuis ſit triens ex 80. hoc eſt 26 $\frac{2}{3}$ Hoc animaduerſo ponamus primum eſſe 1 N, ab eoq; auferamus quæ dat ſecundo $\frac{2}{5}$ N + 6. relinquitur $\frac{1\ 4}{5}$ N — 6. hoc cum ſeptante tertij & 8. æquabitur 26 $\frac{2}{3}$. ergo à 26 $\frac{2}{3}$. auferas $\frac{4}{5}$ N — 6. relinquuntur 32 $\frac{2}{3}$ — $\frac{4}{5}$ N, quod eſt 8. & ſeptans tertij, aufer 8. relinquitur ſeptans tertij 24 $\frac{2}{3}$ — $\frac{4}{5}$ N, ergo tertius eſt 172 $\frac{2}{3}$ — 5 $\frac{3}{5}$ N, huius & primi ſumma 172 $\frac{2}{3}$ — 4 $\frac{3}{5}$ N, detracta à 80. vt pote ſumma trium numerorum, relinquet ſcilicet ſecundum 4 $\frac{3}{5}$ N — 92 $\frac{2}{3}$. huic ſextantem ſuum & 7. adimemus, nimirum $\frac{2}{1}\frac{1}{0}$ N — 8 $\frac{4}{9}$ relinquentur $\frac{2\ 1}{6}$ N — 84 $\frac{2}{9}$ his ſi addamus $\frac{2}{5}$ N + 6: quod ei à primo accedebat, fient $\frac{1\ 2\ 1}{1\ 0}$ N — 78 $\frac{2}{9}$. æqualia

$26\frac{2}{3}$. ob cauſam ſupra demonſtratam, adde vtrobiq; $78\frac{2}{9}$ erunt [illegible] N, æquales [illegible]. hoc eſt 363 N, æquantur 9440, [alterum per 30. alterum per 9. erat multiplicandum, pro his ſume minimos 10. & 3. vt alibi docui] fit 1 N [illegible] primus, ſecundus [illegible] tertius ergo [illegible] quorum ſumma [illegible] ideſt 80. cætera omnia congruere ad poſtulata quæſtionis experiendo deprehendes.

Zeteſis Bacheti.

VErum longe facilius ad vitandas fractionum moleſtias inſtituentur poſitiones hac arte; Eſto tertius 7 N. hic ergo dando ſui ſeptantem + 8. remanebit 6. N − 8. quod cum ſextante ſecundi & 7. æquatur $26\frac{2}{3}$. Quare à $26\frac{2}{3}$ auferendo 6 N − 1. remanet ſextans ſecundi, nimirum $27\frac{2}{3}$ − 6 N. ergo ſecundus eſt 166 − 36 N. qui amiſſo ſextante & 7. remanet $131\frac{1}{3}$ − 30 N. quod cum quintante primi & 6. æquatur $26\frac{2}{3}$ Quare ſi à $26\frac{2}{3}$ detrahas $137\frac{1}{3}$ − 30 N. remanet quintans primi, nimirum 30 N − $110\frac{2}{3}$. eſt ergo primus 150 N − $553\frac{1}{3}$. Reſtat vt trium ſumma conficiat 80. at conficit 121 N − $387\frac{1}{3}$. Hęc ergo æqualia ſunt 80. & fit 1 N. [illegible] igitur tertius qui poſitus erat 7 N. erit [illegible] & alij qui prius.

Zeteſis Glorioſi.

STatuemus primum 5 N. à quo ſi dematur 1 N + 6. hoc eſt quintans & amplius 6. ſupererunt 4 N − 6. quibus à $26\frac{2}{3}$ deductis, hoc eſt à triente numeri dati 80. erit reliquus $32\frac{2}{3}$ − 4 N & hic eſt ſeptans tertij & prętерea 8. habet itaq; primus acceptis & datis $26\frac{2}{3}$ tantundem ergo reliqui, etiam dato receptoque quod imperabatur, habebunt;

bunt; Abs. $32\frac{2}{3}$ — 4 N. ſublatis 8. quod reſtat $24\frac{2}{3}$ — 4N. ſeptans erit tertij, eſt ergo tertius $172\frac{2}{3}$ — 28 N. qui multatus ſui ſeptante & amplius 8. habebit ſummam 140 — 24 N. quæ à triente præſcripto ſubtracta derelinquet 24 N — $113\frac{1}{3}$ puta ſextantem ſecundi & prӕterea 7. & hic quoque tertius acceptis & datis habet $26\frac{2}{3}$. Demum ſi ſumma primi & tertij $172\frac{2}{3}$ — 23 N. à numero præſcripto 80. deducatur, proſiliet ſecundus. 23 N — $92\frac{2}{3}$ qui acceptis & datis vt reliqui poſſidebit $26\frac{2}{3}$ cum itaq; ſextans ſecundi ſublatis 7. ſit 24 N — $120\frac{1}{3}$. erit ſecundus ipſe 144 N — 722. ſed fuit quoque adinuentus 23 N — $92\frac{2}{3}$. Quare 144 N — 722. æquales erunt 23 N — $92\frac{2}{3}$ & reducta æquatione 363 N. æquabuntur 1888. & 1 N. valet [illegible]. eſt ergo primus [illegible] ſecundus [illegible] tertius [illegible].

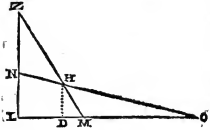
EXER-

EXERCITATIO NONA.

IOannes Terentius Germanus amicus noſter anno 1610. Romæ ad me literas dedit [eram tunc temporis Tridenti] in quarum calce mihi propoſuit duo Problemata ad reſoluendum, Zeteticum vnum, Analyticum alterum.

Problema Zeteticum.

TRianguli rectanguli Z L M latus Z L eſt 12. L M 8. L M protractum eſt ad O, eſtque M O 20. quæritur quanta ſit recta O N bifariam ſecans triangulum Z L M.



Eſto NL, 1 N, ergo triangulum N L O erit 14. N à quo ſi auferatur trapezium N H L M. 24. remanebit triangulum HMO 14 N — 24. à puncto H ducatur HD parallela ad Z L, eaq; ſit 1 A. ergo triangulum HMO erit 10 A, quare 14 N — 24. æquabitur 10 A, & æquatione ordinata.

1 N valebit. $\frac{5}{7}$ A + $\frac{12}{7}$.

Sed vt ZL ad LM, ſic eſt HD ad DM, erit igitur DM $\frac{2}{3}$ A. & ipſa DO $\frac{2}{3}$ A + 20. eſtque N L ad H D vt L O ad D O,

ergo

ergo quod fit ex N L in D O, æquale erit ei, quod fit ex HD in LO; Quare $\frac{10}{21}$ A q $+ \frac{2268}{147}$ A $+ \frac{240}{7}$ æquabitur 28A. ducãtur omnia in 147. & 1848 A − 70 Aq ęquabitur 5040. ergo $26\frac{2}{5}$ A − 1 Aq æquabitur 72. &

1 A valebit $\frac{66}{5} - \sqrt{\frac{2556}{25}}$

Cum itaque 1 N. ipſi $\frac{5}{7}$ A $+ \frac{12}{7}$ atque 1 A ipſi $\frac{66}{5}$

$- \sqrt{\frac{2556}{25}}$ æqualis ſit.

1 N. æquabitur $\frac{66 - \sqrt{2556}}{7} + \frac{12}{7}$

hoc eſt

$11\frac{1}{7} - \sqrt{52\frac{8}{49}}$

& tanta eſt recta N L, ergo quæſita O N erit

Lv $\frac{8495 - \sqrt{62202816}}{49} + \frac{38560}{49}$

hoc eſt

Lv $960\frac{16}{49} - \sqrt{25907\frac{109}{2401}}$

Problema Analyticum.

SI 1. ccc + 16. Qcc + 41. QQc − 23024. cc − 18364. Qc − 133000. QQ − 54505. c + 3728. Q + 8064. N. æquetur 4608. Quæritur valor 1 N.

Cum itaque ordinata Analyſi, inter magnitudines omnes magis prorumpat in anteriora coefficiens ſolidum, pluraque puncta poſſideat conditionaria, quã cubo cubocubus ipſe adfectus, Argumentum eſt adfectum cubocubo-cubum Acephalum eſſe, itaq; vt reſolutioni ſit locus prę-

pręponetur mutilo propoſito cubo - cubo - cubo ea numeralium circulorum multitudo, vt tot puncta cubo - cubo - cubica poſſint ei præfigi, quot cubica ſolido coefficienti, & educta è ſolido coefficiente tanquam cubo radix conſtituetur latus ſingulare primum ipſius reſoluẽdi ita mixtim adfecti cubo - cubo - cubi.

Ex numero igitur propoſito 4608. vt eliciantur latera reſoluendi cubo - cubo - cubi mixtim adfecti, idem arguẽte geneſi erit omnino proceſſus, qui eſſet in Analyſi cubo-cubo - cubi puri, eo tamen addito, vt cõſiderentur omnia, quæ magnitudinibus affirmatis & negatis iuxta ipſarum gradus conueniunt, eaq; rite digerantur ſecundum artem, vt in Paradigmate.

+ coefficiens l	•	ſub quad. cub. cubica
+ coefficiens	•	ſub quad. quad. cubicum
− coefficiens ſ	•	ſub cubo cubicum
− coefficiens p	•	ſub quad. cubicum
− coefficiens p	•	ſub quad. quadraticum
− coefficiens ſ	•	ſub cubicum
+ cofficiens pl	•	ſub quadraticum
+ coefficiens p	•	ſub laterale
c c cubus adfect	8 • cc 2	N. 0 0 / 2 4
Solido ſolido ſol		c c c lateris primi à qcc lat. primi in coeff. longitudinē à qqc lat.primi in coeff. planum. à q.lat.pr. in coef.plan.plan.ſolidum à lat.primo in coeff.plan.ſolid.ſolidū
Summa ablati		

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Solido ſolido ſo					à cc lat.pr.in coeff.ſolid. à qc lat.pr.in coeff. pl.plan. à qq lat.pr.in coeff pl.ſolid. à c lat.pr.in coeff.ſol.ſol.
Summa additit	0				
Exceſſus additi	7	5	2		
Reliquum reſti	2	1	2	8	

+ Pla.ſol.ſol.e				
+ Pla.pla.ſol.				
+ Solid.ſolid.				
+ Plan.ſolid.e				
+ Plan.planũ	8			
+ Solidum exp	9	2		
+ Planum exp	2	5	6	
+ Coefficiens			1	6
+ Pla. ſol ſolid				
+ Pla.pla.ſolid				
+ Sol.ſolid.ex				
Pla. ſolid. ex	0			
+ Pla. planum	4	4		
+ Soli dum ex	5	7	4	
+ Coefficiens			4	1

Diuiſorum pars ſuperior

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— Pla.ſol.ſol.				
— Pla.pla.ſol.				
— Solid. ſolid	o			
— Plan.ſolid.	4	o		
— Plan.planũ	2	8	8	
— Coefficiens	3	o	2	4
— Pla.ſol.ſoli				
— Pla.pla.ſol	o			
— Sol.ſolid.e	6	•		
— Pla.ſolid.e	6	4	o	
— Coefficien	8	3	6	4
— Pla.ſol.ſoli	o			
— Pla.pla.ſol	o	o		
— Solid.ſol.e	o	o	o	
— Coefficien	3	o	o	o
— Pla.ſol.ſol	6	o		
— Pla.pla.ſol	o	3	o	
— Coefficien	4	5	o	5
+ Pla.ſol.ſol.	9	1	2	
+ Coefficien	3	7	2	8
+ Coefficien	8	o	6	4
Reliquum reſt	2 c	1 q	2 N	8 . ccc 2

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Diuiſor
pars
inferi

Summa diu
Summa diu
Exceſſus di

latere ſecundo in 9. qcc. lateris primi
lat. ſecundi q. in 36, qqc. lateris primi
lat. ſecundi c. in 84. cc.lateris primi
Solido ſoli lat. ſecundi qq. in 126. qc.lateris primi
Ablatitia lat. ſecundi qc. in 126. qq.lateris primi
lat. ſecundi cc. in 84. c.lateris primi
lat.ſecundi qqc. in 36. q.lateris primi
lat. ſecundi qcc. in 9. latus primum.
c c lateris ſecundi

latere ſecundo in coeff. plano ſolido ſolidum.
lat. ſecundo in plano ſolido ſolidum explet.
lat. ſecundi q. in coefficiens plano pla. ſolidum
lat. ſecundo in plano ſolido ſolidum explet.
lat. ſecundi q. in plano plano ſolidū explet.
lat. ſecundi c. in ſolido ſolidum explet.
lat. ſecundi qq. in plano ſolidum explet.
lat. ſecundi qc. in plano planum explet.
lat. ſecundi cc. in ſolidum explet.
lat. ſecundi qqc. in coefficiens planum
lat. ſecundo in plano ſolido ſolidum explet.
lat. ſecundi q. in plano plano ſolid. explet.
lat. ſecundi c. in ſolido ſolidum explet.
lat. ſecundi qq. in plano ſolidum explet.
lat. ſecundi qc. in plano planum explet.
lat. ſecundi cc. in ſolidum expletionis
lat. ſecundi qqc. in planum expletionis
lat. ſecundi qcc. in coefficientem longitudinē.

Summa

à lat. ſecundo in plano ſolido ſolidum explet.
à lat. ſecundi q. in plano plano ſolidum explet.
à lat. ſecundi c. in ſolido ſolidum explet.
Solido à lat. ſecundi qq. in plano ſolidum explet.
addititi à lat. ſecundi qc. in plano planum explet.
à lat. ſecundi cc. in coefficiens ſolidum

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lat. ſecundo in plano ſolido ſolidum explet.
lat. ſecundi q. in plano pla. ſolid. explet.
lat. ſecundi c. in ſolido ſolidum explet.
lat. ſecundi qq. in plano ſolidum explet.
lat. ſecundi qc. in coefficiens plano planum

lat. ſecundo in plano ſolido ſolidum explet.
lat. ſecundi q. in plano pla. ſolidum explet.
lat. ſecundi c. in ſolido ſolidum explet.
lat. ſecundi qq. in coefficiens plano ſolidum

lat. ſecundo in plano ſolido ſolidum explet.
lat. ſecundi q. in plano plano ſolidum explet.
lat. ſecundi c. in coefficiens ſolido ſolidum.

Summ

Exceſſ
liqu
cubo a

ITaque ſi 1 ccc + 16qcc. + 41 qqc – 23024 cc – 18364 qc – 133000 qq – 54505 c + 3728 q + 8064 N æquetur 4608. fit 1 N. 24. ex retrograda, quæ omnino obſeruata cernitur compoſitionis via.

E X A M E N.

1. N. valet	24
1. Q.	576
1. c.	13824
1. QQ.	331776
1. Qc.	7962624
1. cc.	191102976
1. qqc.	4586471424
1. Qcc.	110075314176
1. ccc.	2641807540224

Q V A R E.

8064. N. valebunt	193536
3728. Q.	2147328
54505. c.	753477120
133000. QQ.	44126208000
18364. Qc.	146225627136
23024. cc.	4399954919424
41. QQc.	188045328384
16. Qcc.	1761205026816

Addantur ſimilia.

+	1. ccc.	2641807540224
+	16. Qcc.	1761205026816

M + 41.

+	41. qqc.	188045328384
+	3728. Q.	2147328
+	8064. N.	193536
+	Summa	4591060236288
−	23024. cc.	4399954919424
−	18364. Qc.	146225627136
−	133000. QQ.	44126208000
−	54505. c.	753477120
−	Summa	4591060231680

Fiat ſubtractio.

+	Summa	4591060236288
−	Summa	4591060231680
	Numerus propoſitus	4608

EXER.

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EXERCITATIO DECIMA.

IOannes Broſcius Polonus Cracouiēſis Gymnaſij Mathematicus amicus noſter, cum eſſet Patauij, ſignificauit mihi lapſum fuiſſe Petrum Ramum, dum ſcribit ex iſoperimetris homogeneis ordinatius eſſe maius, de qua re ſchedulam mihi talem obtulit.

Petrus Ramus lib. 4. c. 11.

EX iſoperimetris homogeneis ordinatius eſt maius, ex heterogeneis ordinatis terminatius.

In explicatione elementi addit, ordinatius in theoremate intelligatur pro minus inordinato, ſic triangulum æquilaterum maius æquicruro iſoperimetro, & æquicrurum vario; Hoc poſterius [æquicrurum vario] non eſt *κατά παντός*, nam ſi proponantur duo triangula, vnum æquicrurum, 10. 20. 20. alterum varium 11. 18. 21. manifeſtum eſt per Geodæſiam trianguli, poſterius quamuis varium ſit, maiorem habere capacitatem, nimirū ℞. 9800. cum prius habeat ℞ 9375. Addendum itaque aliquid eſt theorematı, vt ſit *καθόλου πρῶτον*, Quamobrem Petrus Ramus hic non eſt *λογικώτατος*, vt eum appellat Franciſcus Vieta; Id propoſitum à me fuit Adriano Romano, extant epiſtolæ ea de re, ſed exemplar nullum ſumpſi.

Per hanc ſchedulam inuitauit me ad contemplandum, quomodo ſuppleri debeat defectus theorematis, vt amico itaque morem geram, aliquid dicere non grauabor, proponam contemplationem Geodætice per theoremata.

THEOREMA I.

EX iſoperimetris homogeneis ordinatum eſt maius.

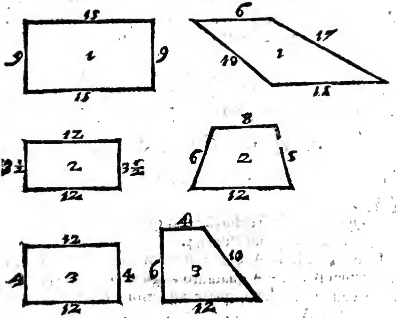
Proponantur triangula, æquilaterum $16\frac{2}{3}$. $16\frac{2}{3}$: $16\frac{2}{3}$ æquicrurum 10. 20. 20. & varium 11. 18. 21. erit area trianguli æquilateri ℞ $14467\frac{1}{2}-\frac{6}{7}$. area æquicruri, ℞ 9375. & area varij ℞ 9800. at maior eſt ℞ $14467\frac{1}{2}-\frac{6}{7}$ quam ℞ 9375. & quam ℞ 9800. maius eſt igitur triangulum æquilaterum quam æquicrurum & quam varium iſoperimetrum.

THEOREMA II.

EX iſoperimetris homogeneis inordinatis, fieri poteſt quod minus inordinatum ſit maius, minus & æquale.

Proponantur tria oblonga & tria trapezia, latera primi oblongi 15. 15. 9. 9. ſecundi 12. 12. $3\frac{1}{2}$ $3\frac{1}{2}$. tertij 12. 12. 4. 4. latera primi trapezij 15. 6. 10. 17. ſecundi 12. 8. 6. 5. tertij 12. 4. 6. 10. vt ex appoſitis apparet figuris, erit area primi oblongi 135. ſecundi 42. tertij 48. area vero primi trapezij 84. ſecundi ℞. $2460\frac{1}{2}$ – [illegible], tertij 48. maius eſt igitur primum oblongum, ſecundum verò minus, at tertium prorſus æquale ſuo compari trapezio iſoperimetro, attamen minus inordinatum eſt oblongum quam trapezium.

THEO-



THEOREMA III.

Ex iſoperimetris heterogeneis ordinatis terminatius eſt maius.

Proponantur triangulum ęquilaterum, cuius latus $16\frac{2}{3}$ & quadratum, cuius latus $12\frac{1}{2}$ erit area trianguli æquilateri ℞ $14467\frac{1}{3}\frac{8}{9}$ & area quadrati $156\frac{1}{4}$ at maior eſt $156\frac{1}{4}$ quam ℞ $14467\frac{1}{3}\frac{8}{9}$. maius eſt igitur quadratum triangulo æquilatero iſoperimetro.

THEOREMA IV.

Ex iſoperimetris heterogeneis inordinatis, fieri poteſt quod terminatius ſit maius, minus & æquale.

Propo-

Proponantur triangulum varium 10. 17. 21. & tria oblonga, primi latera circa rectum 14. 10. ſecundi 20. 4. tertij ℞ 60 + 12. 12. − ℞ 60. erit area trianguli varij 84. & area primi oblongi 140. ſecundi 80. tertij 84. eſt igitur oblongum primum maius, ſecundum vero minus, at tertium prorſus æquale triangulo vario iſoperimetro, attamē terminatius eſt oblongum quam triangulum.

SCHOLIVM.

AD inueniendum latera tertij oblongi, hoc vſus ſum artificio.

Oportet inuenire duo latera, quorum ſumma ſit 24. & productum 84.

Eſto maius latus A, ergo reliquum erit B − A, horum productum B in A − A quadrato, igitur B in A − A quadraro æquatur D plano, quare æquatione ordinata

B in A − D plano	æquabitur	A quadrato

ſit B 24. D planum 84. & ſit A latus maius, ℞ 60 + 12, latus minus 12. − ℞ 60.

THEOREMA V.

EX iſoperimetris heterogeneis partim ordinatis partim inordinatis, terminatius ordinatum eſt maius.

Proponantur triangulum varium & quadratum, per tertium Theorema quadratum eſt maius triangulo æquilatero iſoperimetro, at per primum triangulum æquilaterum maius vario iſoperimetro, quadratum igitur multo magis triangulo vario iſoperimetro maius erit; Quod ſi terminatius

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tius non fuerit ordinatum, fieri poteſt quod ſit maius ordinato iſoperimetro, minus & æquale, vt in antecedente theoremate determinatũ eſt de iſoperimetris inordinatis.

CONTEMPLATIO.

THeorema primum & tertium demonſtrarũt veteres, Theon præſertim & Pappus, ſecundũ vero & quartum Clauius lib. 7. Geometriæ practicæ Prop. 22. dum praxim tradit, quomodo dato rectilineo parallelogrammum rectangulum iſoperimetrum maius, minus & æquale dari poſſit, nam ſi rectilineum illud fuerit inordinatum, at homogeneum cum parallelogrammo rectãgulo, eius problema non differt à ſecundo noſtro Theoremate, ſed ſi rectilineum illud fuerit inordinatum, at heterogeneum cum parallelogrammo rectaagulo, eius problema non differt à quarto noſtro Theoremate, quintum notum eſt ex ſuperioribus; Tamen adhuc à nemine Geometrice demonſtratum eſt [quod ſciam] quomodo datis duabus figuris iſoperimetris inordinatis, ſiue homogeneis ſiue heterogeneis, cognoſcere poſſimus quæ illarum maior ſit, fortaſſis, quia animaduerterunt nihil certi determinari poſſe, equidem regulam nullam generalem inuenire potui: Sed redeundo ad noſtrum propoſitum de figuris homogeneis inordinatis, in primis conſideraui, num aliquid conferret baſium æqualitas, in triangulis verum eſt & à veteribus demonſtratum, hoc eſt ſi duo triangula, æquicrurum videlicet & varium ſuper eandem baſim conſtituta fuerint, æquicrurum maius eſſe vario iſoperimetro; Verum quoque eſt in parallelogrammis, nam ſi ſuper eandem baſim conſtituantur duo parallelogramma, rectangulum & obliquãgulum, rectangulum obliquãgulo iſoperimetro maius erit, at in quadrilateris vniuerſim nequaquam verum eſt, ete-

nim

nim ſi acceperimus duo quadrilatera, oblongum & trapezium, æqualem baſim habentia, quamuis oblongum minus ſit inordinatum quam trapezium, fieri tamen poteſt, quod oblongum maius, minus & æquale ſit ipſi trapezio iſoperimetro, vt in ſecundo Theoremate à nobis oſtenſum eſt; Tandem conſideraui quod ſi regula & norma in homogeneis eſt ordinatum, hoc eſt figura æquilatera & ęquiangula, quæ omnium iſoperimetrarum eſt capaciſſima, homogeneum illud, quod magis accedit ad ordinatum, capacius erit, hinc tale concepimus theorema.

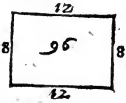
THEOREMA.

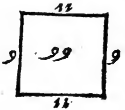
EX iſoperimetris homogeneis inordinatis quod magis accedit ad ordinatũ eſt maius.

Hoc theorema quatuor caſus habere poteſt, 1. quando homogenea conueniunt in angulis & diſcrepant in lateribus, 2. quando conueniunt in lateribus & diſcrepant in angulis 3. quando conueniunt in angulis & in lateribus 4. quando diſcrepant in angulis & in lateribus.

PRIMVS CASVS.

QVando homogenea conueniunt in angulis & diſcrepant in lateribus, tunc homogeneũ illud maius erit, cuius latera magis accedunt ad latera ordinati; Proponantur duo oblonga iſoperimetra, latera primi 12.12.8.8. latera ſe-

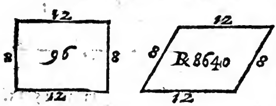




ſecundi 11. 11. 9. 9. cum itaque oblonga habeant ſingulos angulos æquales, nempe rectos, quos etiam poſſidet ordinatum, nempe qnadratum, & latera ſecundi oblongi magis accedant ad latera quadrati, latus quadrati iſoperimetri eſt 10. latera igitur ſecundi oblongi diſtant à latere quadrati per 1. at latera primi per 2. maius igitur eſt ſecundum oblongum quam primum iſoperimetrum, area primi oblongi eſt 96. area ſecundi 99.

SECVNDVS CASVS.

QVando homogenea conueniunt in lateribus & diſcrepant in angulis, tunc homogeneum illud maius erit, cuius anguli magis accedunt ad angulos ordinati; Proponãtur duo parallelogramma iſoperimetra, rectangulum vnum, obliquangulum alterum, latera



vtriuſque eadem 12. 12. 8. 8. tamen obliquanguli diameter ſit 16. cum itaque hæc duo parallelogramma habeant ſingula latera æqualia, & anguli parallelogrammi rectanguli magis accedant ad angulos quadrati, imo eoſdem poſſidet angulos, quos poſſidet quadratum, nempe rectos, maius igitur eſt parallelogrammum rectangulum quam obliquangulum iſoperimetrum, area rectanguli eſt 96. area obliquanguli ℞ 8640.

TERTIVS CASVS.

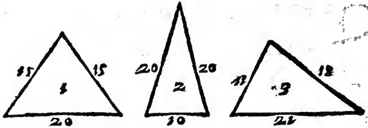
QVando homogenea conueniunt in angulis & in lateribus, tunc prorſus ſunt iſoperimetra & æqualia,

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nam cum ſingula latera vnius adæquentur ſingulis lateribus alterius,& ſinguli anguli ſingulis angulis,quid amplius requiritur pro æqualitate ?

QVARTVS CASVS.

QVando homogenea diſcrepant in angulis & in lateribus, tunc homogeneum illud maius erit,cuius latera & anguli magis accedunt ad latera & angulos ordinati ; Proponantur tria triangula iſoperimetra,



duo æquicrura & varium, primum æquicrurum 20.15. 15. ſecundum 10. 20. 20. varium 21. 11. 18. anguli primi equicruri gr. 83.36. gr. 48. 12. gr.48.12. anguli ſecundi æquicruri gr.28.58. gr.75.31. gr. 75.31. anguli ipſius varij gr.89.28. gr.57. 32. gr.33,0.cum itaq; latera & anguli primi ęquicruri magis accedant ad latera & angulos ordinati, hoc eſt trianguli æquilateri, quam latera & anguli ipſius varij, & latera & anguli ipſius varij magis quam latera & anguli ſecundi æquicruri, vt ex adnotatis apparet differentijs, maius igitur erit primum ęquicrurum vario,& varium ſecundo æquicruro iſoperimetro, area primi æquicruri ℞.12500. area ſecundi ℞ 9375.area varij ℞ 9800.

Nota

Notatio laterum & angulorum.

Pro primo æquicruro.

$16\frac{2}{3}$.	$16\frac{2}{3}$.	$16\frac{2}{3}$.	latera æquilateri
15.	15.	20.	latera primi æquicruri.
$1\frac{2}{3}$	$1\frac{2}{3}$	$3\frac{1}{3}$.	differentiæ.
60-0.	60-0.	60-0.	anguli æquilateri.
48-12.	48-12.	83-36.	anguli primi æquicruri.
11-48.	11-48.	23-36.	differentiæ.

Pro ſecundo æquicruro.

$16\frac{2}{3}$.	$16\frac{2}{3}$.	$16\frac{2}{3}$.	latera æquilateri.
20.	20.	10.	latera ſecundi æquicruri.
$3\frac{1}{3}$.	$3\frac{1}{3}$.	$6\frac{2}{3}$.	differentiæ.
60-0.	60-0.	60-0.	anguli æquilateri.
75-31.	75-31.	28-58.	anguli ſecundi æquicruri.
15-31.	15-31.	31-2.	differentiæ.

Pro vario.

$16\frac{2}{3}$.	$16\frac{2}{3}$.	$16\frac{2}{3}$.	latera æquilateri.
11.	21.	18.	latera varij.

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$5\frac{2}{7}$.	$4\frac{1}{3}$.	$1\frac{1}{6}$.	differentiæ.
60-0.	60-0.	60-0.	anguli æquilateri.
57-32.	33-0.	89-28.	aoguli varij.
2-28.	27-0.	29-28.	differeutiæ.

Pro primo æquicruro.

$6\frac{2}{7}$. differentia laterum.

47-12 differentia angulorum.

Pro ſecundo æquicruro.

$13\frac{2}{3}$. differentia laterum.

62-4. differentia angulorum.

Pro vario.

$11\frac{1}{3}$. differentia laterum.

58-56. differentia angulorum.

S C H O L I V M.

QVod ſi forte in vno homogeneo latera minus diſtent à lateribus ordinati, at anguli magis ab angulis, quam in altero, vel contra, tunc nihil certi determinari poteſt, nam modo vnum, modo alterum maius erit.

De

De Theoremate Rami

THeorema Rami nullo indiget ſupplemento, niſi quod verbum ordinatius alio modo gloſſandum eſt, quam ipſe fecit, equidem Ramus exiſtimauit triangulum æquicrurum ordinatius eſſe quam varium, quia duos poſſidet angulos æquales ac duo æqualia latera, at nos ordinatius appellamus illud triangulum, cuius latera & anguli magis accedunt ad latera & angulos trianguli æquilateri, ſiue æquicrurum illud ſit ſiue varium; Tamen ſi ad clariorem doctrinam theorematí aliquid addendum eſſet, vtiq; illud ſic proponendum eſſe exiſtimarem.

Ex iſoperimetris homogeneis vniuerſis ordinatum eſt maius, ex inordinatis ordinatius, hoc eſt quod magis accedit ad ordinatum.

An vero hæc contemplatio à nobis habita perfecta & abſoluta dici debeat, nullam habeo certitudinem, rationabile tamen eſt homogeneum illud maius & capacius confiteri, quod eſt ordinatius, hoc eſt quod magis accedit ad figuram iſoperimetram æquilateram & æquiangulam, ſufficit tamen oſtendiſſe, & ſecundum hanc contemplationem aliquam rationem aſſignaſſe, cur triangulum varium 21. 11. 18. maius ſit æquicruro 10. 20. 20. at minus æquicruro 20. 15. 15. iſoperimetro. An igitur hæc ita ſe habeant in omnibus triangulis, & ſi ita ſe habeant in omnibus triangulis, ita quoque ſe habeant in omnibus reliquis figuris, affirmare non audeo, nulla enim id Geometrica ratione adhuc confirmatum eſt, libenter viderem quæ ab ipſo Broſcio & ab Adriano Romano ſuper hac re conſcripta ſunt.

FINIS.

Pag.	Lin.	Errata	Corretta.
7	4	in mores	minores
22	9	G I K	G I H
	11	erar	erat
59	5	lunarum	lunularum
61	3	C G L G	C G L F
	8	— 5647112	— ℞ 5647112
	16	potaſtates	poteſtates
65	26	æqualæ	æquale
67	28	C G F L	C G L F
91	10	in 5. c. latus primum	in 5. latus primum
	19	Pla. ſol. ſol. ſol.	Pla. ſol. ſol.
	21	Coefficiens plano ſolido ſolidum	Coefficiens plano plano ſolidum

Pag. 38. figura 2. deeſt litera G, collocanda in concurſu rectarum A F, B D.

Pag. 58. in figura deeſt litera R, collocanda in decuſſatione rectarum C D, A L.

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